section -1 Arithmetical Ability

(1) operations on numbers:-

* numbers. In hindu- arabic system, we have ten digits, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and called zero, one, two, three, , and respectively.

A number is denoted by a group of digits, called numerals.

For denoting a numeral, we use the place-value chart, given below.

Ex.1 write each of the followings numeral in words

	Ten crores	crores	Ten lacs	lacs	Ten thou	thou	Hund	Ten	units
(i)				6	3	8	5	4	9
(ii)			2	3	8	0	9	1	7
(iii)		8	5	4	1	6	0	0	8
(iv)	5	6	1	3	0	7	0	9	0

sol:- the given numerals in words are

(i). Six lac thirty-eight thousand five hundred forty nine.

(ii). Twenty-three lac eighty thousand nine hundred seventeen.

(iii) Eight crore fifty-four lac sixteen thousand eight.

(iv) Fifty six crore thirteen lac seven thousand ninety

Ex.2 write each of the following numbers in figures.

(i) nine crore four lac six thousand two

(ii) Twelve crore seven lac nine thousand two hundred seven.

∴ 246591 is divisible by 9.

(ii). In the number 734519, the sum of digits = 29, which is not divisible by 9.

∴ 734519 is not divisible by 9.

(iv) Divisibility by 4 :-

A number is divisible by 4 is the sum of its last two digits is divisible by 4.

Ex: (i). 6879876 is divisible by 4, since 76 is divisible by 4. (ii) 496138 is not divisible by 4, since 38 is not divisible by 4.

(v) Divisibility by 8 :-

A number is divisible by 8 if the number formed by hundred's ten's and units digit of the given number is divisible by 8.

Ex: (i). In the number 1678952, the number formed by last 3 digits, namely 852 is divisible by 8.

∴ 1678952 is divisible by 8

(ii). In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.

∴ 576484 is not divisible by 8.

(vi) Divisibility by 10 :-

A number is divisible by 10 only when its unit digit is 0

Ex: (i). 7849320 is divisible by 10, since its unit similarly, the H.C.F of more than three numbers may be obtained.

(iii). Least common multiple L.C.M: The least number which is exactly divisible by each one of the given numbers is called the L.C.M.

common division method to find
(2). ~~Factorisation~~ Method of finding L.C.M:- Arrange the given numbers in a row in any order. Divide by a number which divides exactly a least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is its required L.C.M of the given numbers.

(1) Factorisation method of finding L.C.M:- Resolve each one of the given number into a product of prime factor. Then L.C.M is the product of highest powers of a the factors.

(iv) Product of two numbers = product of their H.C.F and L.C.M.

(v). Co- primes: Two numbers are said to be co-primes if their H.C.F is 1

(vi). H.C.F and L.C.M of fractions:-

(1) $\text{H.C.F} = \dfrac{\text{H.C.F of numerators}}{\text{L.C.M of denominators}}$

(2). $\text{L.C.M} = \dfrac{\text{L.C.M of numerators}}{\text{H.C.F of denominators}}$.

suppose we have to find the quotient (0.0204 ÷ 17).
now, 204 ÷ 17 = 12. Dividend contains 4 places of decimal. so 0.0204 ÷ 17 = 0.0012.

(5). Dividing a decimal fraction by a decimal fraction: multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. now, proceed as above.

Thus, $\frac{0.0066}{0.11} = \frac{0.0066 \times 100}{0.11 \times 100} = \frac{0.066}{11} = 0.006$.

(V). comparison of fractions:- suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly

suppose, we have to arrange the fractions $3/5$, $6/7$ and $7/9$ in descending order.
now, $3/5 = 0.6$, $6/7 = 0.857$, $7/9 = 0.777$
Since $0.857 > 0.777 > 0.6$ so $6/7 > 7/9 > 3/5$.

(VI). Recurring decimal:- If in a decimal fraction, a figure or set of figures is repeated continuously, then such a number is called a recurring decimal.
In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is

Solved examples

Ex: 1 $9587 - ? = 7429 - 4358$

Sol:- Let $9587 - x = 7429 - 4358$, then,

$9587 - x = 3071 \Rightarrow x = 9587 - 3071 = 6516$

Ex: 2 $5793405 \times 9999 = ?$

Sol:

$5793405 \times 9999 = 5793405 \times (10000 - 1)$

$= 57934050000 - 5793405$

$= 57928256595$

Ex: 3 $839478 \times 625 = ?$

Sol:- $839498 \times 625 = 839498 \times 5^4$

$= 839478 \times \left(\frac{10}{2}\right)^4 = \frac{839478 \times 10^4}{2^4}$

$= \frac{8394780000}{16} = 524678750$

Ex: 4 $976 \times 237 + 976 \times 763 = ?$

Sol:- using distributive law, we get

$976 \times 237 + 976 + 763 = 976 \times 1237 + 763$

$= 976 \times 1000 = 976000$

Ex: 5 $986 \times 307 - 986 \times 207 = ?$

Sol:- By distributive law, we get

$986 \times 307 - 986 \times 207 = 986 \times (307 - 207)$

$= 986 \times 100 = 98600$

Ex: 6 $1607 \times 1607 = ?$

$1607 \times 1607 = (1607)^2$

at whatever place it may be.

Ex:3 In the numeral 8184925, write down;

(i). Face value of 1 (ii). Face value of 9 (iii) place value of 4

(iv) Place value of 3 (v) Place value of 8 (vi) place value of 5

Sol:- writing the given numeral in place-value chart, we get,

Ten lacs	lacs	Ten thou	thou	Hund	tens	ones
8	1	3	4	9	2	5

(i). Face value of 1 is 1

(ii) Face value of 9 is 9

(iii) place value of of 4 $=(4\times 1000) = 4000$

(iv) place value of 3 $=(3\times 10000) = 30000$

(v) place value of 8 $=(8\times 1000000) = 8000000$

(vi) place value of 5 $=(5\times 1) = 5$

various types

(i). natural numbers: counting numbers are called natural numbers.

(ii) whole numbers:- All counting numbers and 0 from the set of whole numbers. Thus 0, 1, 2, 3, 4, 5... etc clearly, every natural numbers is whole number and 0 is a whole number which is not a natural number

(iii). integers:- All counting numbers, zero and negatives of counting numbers from the set of integers Thus, ..., -3, -2, -1, 0, 1, 2, 3... are all integers set of positive numbers = (1, 2, 3, 4, 5, 6 ...)

$= 2560000 + 49 + 22400 = 2582449.$

Ex : 7 $\quad 1396 \times 1396 = ?$

Sol :- $1396 \times 1396 = (1396)^2$

$= (1400-4)^2 = (1400)^2 + 4^2 - 2 \times 1400 \times 4$

$= 1960000 + 16 - 11200 = 1948816.$

Ex : 8 $\quad (475 \times 475 + 125 \times 125) = ?$

Sol :- we have $(a^2+b^2) = \frac{1}{2}\left((a+b)^2 + (a-b)^2\right)$

$\therefore (475)^2 + (125)^2 = \frac{1}{2}\left((475+125)^2 + (475-125)^2\right)$

$= \frac{1}{2}\left[(600)^2 + (350)^2\right] = \frac{1}{2}(360000 + 12250$

$= \frac{1}{2} \times 482500 = 241250$

Ex : 9 $\quad (796 \times 796 - 204 \times 204) = ?$

Sol :- $796 + 796 - 204 \times 204 = (796)^2 - (204)^2$

$= (796+204)(796-204)$

$= (1000 \times 592) = 592000$

Ex : 10 $\quad (387 \times 387 + 113 \times 113 + 2 \times 387 \times 113) = ?$

Sol :- Given exp $= (387)^2 + (113)^2 + 2 \times 387 + 113$

$= (387+113)^2 = (500)^2 = 250000$

Ex : 11 $\quad (87 \times 87 + 61 \times 61 - 2 \times 87 \times 61) = ?$

Sol : Given exp $= (87)^2 + (61)^2 - 2 \times 87 \times 61$

$= (87-61)^2 = (26)^2 = (20+6)^2$

$= 20^2 + 6^2 + 2 \times 20 \times 6 = (400 + 36 + 240)$

sol > (i) we know that $(12)^2 > 137$

prime numbers less than 12 are 2, 3, 5, 7, 11

clearly, none of them divides 137

$\therefore$ 137 is a prime number.

(ii) we know that $(14)^2 > 173$

prime number less than 14 are 2, 3, 5, 7, 11, 13

clearly, none of them divides 173

$\therefore$ 173 is a prime number.

(iii) we know that $(18)^2 > 319$

Prime numbers less than 18 are 2, 3, 5, 7, 11, 13, 17

out of these prime numbers, 11 divides 319 respect..

$\therefore$ 319 is not a prime number.

(iv). we know that $(21)^2 > 437$

prime number less than 21 are 2, 3, 5, 7, 11, 13, 17, 19

clearly, 437 is divisible by 19

$\therefore$ 437 is not a prime number.

(v). We know that $(30)^2 > 811$

prime number less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

clearly, none of those numbers divisible 811

$\therefore$ 811 is a prime number.

composite number :-

The natural numbers which are not prime are called composite numbers.

H.C.F and L.C.M of numbers.

important facts and formulae

(i) Factors and multiples:- If a number a divides another number b exactly, we that a is a factor of b. In this case, b is called a multiple of a.

(ii). H.C.F or G.C.M; G.C.D: The H.C.F of two or more than two numbers is the greatest number that divides each of them exactly.

there are two methods of finding the H.C.F of a given set of numbers.

(i). Factorisation method:- Express each one of the given numbers as the product prime factors. The product of least powers of common prime factors gives H.C.F.

(i). Division method:- suppose we have to find the H.C.F of two given numbers. Devide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F

Finding the H.C.F of more than two numbers: Suppose we have to find the H.C.F of three numbers. then, H.C.F of [(H.C.F of any two) and (the third number gives the HCF of three given numbers.

(iii) Four lac four thousand forty

(iv) Twenty-one crore sixty lac five thousand fourteen.

sol:- using the place value chart, we may write

	Ten crore	crores	Ten lacs	lacs	10 thou	thou	Hund	Tens	units
(i)		9	0	4	0	6	0	0	2
(ii)	1	2	0	7	0	9	2	0	7
(iii)				4	0	4	0	4	0
(iv)	2	1	6	0	5	05	0	1	4

(2) · Face value and place value of (or local value) of a digit in a numeral.

(i) The face value of a digit in a numeral is its own value at whatever place it may be.

Ex: In the numeral 6872, the face value of 2 is 2, the face value of 8 is 8 and the face value of 6 is 6.

(ii). In a given numeral

place value of unit digit = (unit digit) ×1

place value of tens digit = (ten digit) ×10

place value of hundreds digit = (hundreds digit) ×100

Ex: In the numeral 10984, we have

place value 4 = (4 × 1) = 4

place value of 8 = (8 ×10) = 80

place value of 9 = (9 ×100) = 900

place value of 1 = (1 × 10000) = 10,000

note: place value of 0 in a given numeral is 0

(ii) $18800 \div 470 + 20 = \frac{18800}{470} + 20 = 40 + 20 = 2$.

Ex:2 Simplify: $b - [b - (a+b) - [b - (b - \overline{a-b})] + 2a]$.

Sol: Given Expression $= b - [b - (a+b) - (b - (b - a + b)) + 2a]$

$$= b - [b - a - b - (b - 2b + a)) + 2a].$$

$$= b - (-a - (b - 2b + a + 2a))$$

$$= b - [-a - (-b + 3a)] = b - (-a + b - 3a)$$

$$= b - [-4a + b] = b + 4a - b = 4a.$$

Ex:3 What value will replace the question mark in the following equation? $4\frac{1}{2} + 3\frac{1}{6} + ? + 2\frac{1}{3} = 13\frac{2}{5}$

Sol: Let $\frac{9}{2} + \frac{19}{6} + x + \frac{7}{3} = \frac{67}{5}$

Then, $x = \frac{67}{5} - \left(\frac{9}{2} + \frac{19}{6} + \frac{7}{3}\right) \Rightarrow x = \frac{67}{5} - \left(\frac{27+19+14}{6}\right)$

$$= \left(\frac{67}{5} - \frac{60}{6}\right) \Rightarrow x = \left(\frac{67}{5} - 10\right) = 3\frac{2}{5}$$

Hence, missing fraction $= 3\frac{2}{5}$.

Ex:4 $\frac{4}{13}$ of $\frac{3}{7}$ of number is greater than $\frac{4}{9}$ of $\frac{2}{3}$ of the same number by what is half of that number

Sol: Let the number be x, then, $\frac{4}{13}$ of $\frac{3}{7}$ of $x - \frac{4}{9}$ of $\frac{2}{3}$ of $x = 8 \Rightarrow \frac{4}{21}x - \frac{8}{45}x = 8$

$$\Rightarrow \left(\frac{4}{21} - \frac{8}{45}\right)x = 8 \Rightarrow \left(\frac{90-88}{315}\right)x = 8 \Rightarrow \frac{4}{315}x = 8$$

$$\Rightarrow x = \left(\frac{8 \times 315}{4}\right) = 630 \Rightarrow \frac{1}{2}x = 315$$

Hence, required number $= 315$.

3. Decimal fractions

(1). Decimal fractions :- Fractions in which denominators are powers of 10 are known as decimal fractions.

Thus, $1/10 = 1$ tenth $= .1$: $1/100 = 1$ hundredth $= 0.01$:

$99/100 = 99$ hundredths $= 0.99$, $7/1000 = 7$ thousandths $= 0.007$, etc.

(ii). conversion of a decimal into vulgar fractions: put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction of its lowest terms.

Thus, $0.25 = 25/100 = 1/4$; $2.008 = \frac{2008}{1000} = \frac{251}{125}$.

(iii) (1). Annexing zeros to the extreme right of a decimal fraction does not change its value. Thus, $0.8 = 0.80 = 0.800$, etc.

(2) If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.

Thus, $\frac{1.84}{2.99} = \frac{184}{299} = \frac{8}{13}$; $\frac{.365}{.584} = \frac{365}{584} = 5/8$.

(iv) operations of decimal fractions:-

(1). Addition and subtraction of decimal fractions:- The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

(8) co-prime :-

Two natural numbers a and b are said to be co-prime. If their HCF is 1.

Ex: (2,3), (4,5), (7,9), (8,11) etc.. are pairs are of co-primes.

Tests of divisibility

(i) Divisibility by 2 :-

A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

Ex: 58694 is divisible by 2, while 86945 is not divisible by 2.

(ii) Divisibility by 3 :

A number is divisible by 3 only when the sum of its digits is divisible by 3.

Ex (i). In the number 695421, the sum of digits = 27, which is divisible by 1

∴ 695421 is divisible by 3.

(ii). In the number 948653, the sum of digits = 35, which is not divisible by 3.

∴ 948653 is not divisible by 3.

(iii). Divisibility by 9 :-

A number is divisibility by 9 only when the sum of its digits is divisible by 9.

Ex: (i) In the number 246591, the sum of digits = 27, which is divisible by 9.

set of negative integers = (-1, -2, -3 . . .)

set of all non-negative integers = (0, 1, 2, 3, 4 . .)

(4). even and odd numbers.

i). even numbers : A counting number divisible by 2 is called an even number.

(ii) odd number : A counting number not divisible by 2 is called an odd number

thus 1, 3, 5, 7, 9, 11, 13, 15 . . etc are all odd numbers.

(5) prime numbers:-

A counting number is called a prime number if it has exactly two factors namely itself and 1.

Ex:- All prime numbers less than 100 are

2, 3, 5, 7, ~~9~~, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

test for a number to be prime:-

let P be a given number and let n be the smallest counting number such that $n^2 \geq P$. Now, test whether P is divisible by any of the prime numbers less than or equal to n

Ex: 4 Test whether of the following are prime no

(i) 137 (ii) 173 (iii) 319 (iv). 487 (v) 811

Ex:5 simplify : $\left(3\frac{1}{4}-\left(2\frac{2}{4}-\frac{1}{2}\left(2\frac{1}{2}-\frac{1}{4}-\frac{1}{6}\right)\right)\right)$.

Sol:- Given Ex $=\left(\frac{13}{4}-\left(\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{3-2}{12}\right)\right)\right)$

$=\left(\frac{13}{4}-\left(\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{1}{12}\right)\right)\right)=\left(\frac{13}{4}-\left(\frac{5}{4}-\frac{1}{2}\left(\frac{30-1}{12}\right)\right)\right)$

$=\left(\frac{13}{4}-\left(\frac{5}{4}-\frac{29}{24}\right)\right)=\left(\frac{13}{4}-\left(\frac{30-29}{24}\right)\right)=\left(\frac{13}{4}-\frac{1}{24}\right)$

$=\left(\frac{13}{4}\times 24\right)=78$.

Ex:6 simplify : $108 \div 36$ of $\frac{1}{4}+\frac{2}{5}\times 3\frac{1}{4}$.

Sol: Given Exp $=108 \div 9+\frac{2}{5}\times\frac{13}{4}=\frac{108}{9}+\frac{13}{10}=\left(12+\frac{13}{10}\right)$

$=\frac{133}{10}=13\frac{3}{10}$.

Ex:7 simplify $\dfrac{\frac{7}{2}\div\frac{5}{2}\times\frac{3}{2}}{\frac{7}{2}\div\frac{5}{2}\text{ of }\frac{3}{2}}\div 5.25$.

Sol:- Given Exp $=\dfrac{\frac{7}{2}\times\frac{2}{5}\times\frac{3}{2}}{\frac{7}{2}\div\frac{15}{4}}\div 5.25=\dfrac{\frac{21}{10}}{\frac{7}{2}\times\frac{4}{15}}\div\frac{525}{100}$

$=\frac{21}{10}\times\frac{15}{14}\times\frac{100}{525}=\frac{6}{14}=\frac{3}{7}$.

Ex:8 simplify :(i) $12.05\times 5.4 \div 0.6$ (ii) $.6\times.6+.6\div.6$.

Sol:(i) Given Exp $=12.05\times\frac{5.4}{0.6}=12.05\times 9=108.45$.

(ii) Given Exp $=.6\times.6+\frac{6}{6}=.36+1=.46$.

Ex:9 Find the value of x in each of the following eqn:-

(i) $\frac{17.28\div x}{3.6\times 0.2}=2$ (ii). $3648.24+364.824+x-36.4824=3794.168$

(iii). $8.5-\left(5\frac{1}{2}-\left(7\frac{1}{2}\times 2.8\div x\right)\right)\times 4.25\div(0.2)^2=306$.

Sol:- (i) $\frac{17.28}{x}=2\times 3.6\times 0.2 \Rightarrow x=\frac{17.28}{1.44}=\frac{1728}{144}=12$

(ii). $\frac{364.524}{x}=3794.1696+36.4324+3830.652-3648.24$ $+182.412$

$\Rightarrow x=\frac{364.524}{182.412}=2$.

unit - 4 simplification

Important concepts

(I). 'BODMAS' Rule: This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression. Here, 'B' stands for 'Bracket', 'O' for 'of', 'D' for 'Division', 'M' for 'multiplication' 'A' for 'Addition' and 'S' for 'subtraction'.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order ( ), { } and [ ].

After removing the brackets, we must use the following operations strictly in the order.

(i) of (ii) division (iii) multiplication (iv) Add (v) Sub

(II). modulus of real number: modulus of real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a>0 \\ -a, & \text{if } a<0 \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

(III). virnaculum (or Bar): when an expression contains virnaculum, before applying the 'BODMAS' rule, we simply the expression under the virnaculam.

Solved Examples

Ex:1) simplify : (i) $5005 - 5000 \div 10$ (ii) $18800 \div 470 \div 20$

Sol: (i) $5005 - 5000 \div 10 = 5005 - \frac{5000}{10} = 5005 - 500$

$= 4505$.