Operations on numbers:

1. Example: 1. $9587 - ? = 7429 - 4358$.

Let $9587 - x = 7429 - 4358$ then.

$9587 - x = 3071 \Rightarrow x = 9587 - 3071 = 6516$.

2). Example:

$(796 \times 796 - 204 \times 204) = ?$.

$796 \times 796 - 204 \times 204 = (796)^2 - (204)^2$.

$= (796 + 204)(796 - 204) = (1000 \times 592) = 592000$.

3). What least number must be added to 2010 to obtain a number which is completely divisible by 19.

$19 \overline{)2010} (105$

$\underline{19}$

110

$\underline{95}$

15

Number to be added $= (19 - 15) = 4$.

4). Simplify: $\frac{658 \times 658 \times 658 - 328 \times 328 \times 328}{658 \times 658 + 658 \times 328 + 328 \times 328} = ?$.

$\frac{(658)^3 - (328)^3}{(658)^2 + 658 \times 328 + (328)^2} = \frac{a^3 - b^3}{(a^2 + ab + b^2)}$ where $a = 658$ and $b = 328$

$(a - b) = 658 - 328 = 330$

5). What unit digit in product $(684 \times 759 \times 413 \times 676)$.

Unit digit in the product $(4 \times 9 \times 3 \times 6) = 8$.

2. H.C.F and L.C.M of numbers:

1). Find the H.C.F of 108, 288 and 360:

1) $108 = 2^2 \times 3^3 \times 288 - 2^5 \times 3^2$ and $360 - 2^2 \times 6 \times 3^2$

$H.C.C.F = 2^2 \times 8^2 = 36.$

2). Find H.C.F of 513, 1134 and 1215.

2)
$$1134 \overline{)1215} (1$$
$$1134$$
$$81) 1134 (14$$
$$81$$
$$324$$
$$324$$
$$x$$

H.C.F of 1134 and 1215 is 81.

Required H.C.F of 513 and 81.

3)
$$81) 513 (6$$
$$486$$
$$27) 81 (3$$
$$81$$
$$x$$

H.C.F of given number = 27.

3). Reduce $\frac{391}{667}$ to lowest terms:-

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}.$$

4). Find the L.C.M of $2^2 \times 3^3 \times 5 \times 7^3 \times 8^2 \times 3^2 \times 5^2 \times 7^2$ $2 \times 3 \times 3^2 \times 7 \times 11.$

L.C.M = product of highest powers 2, 3, 5, 7 and 11 $= 2^8 \times 3^3 \times 5^2 \times 7^2 \times 11$

5). Find the L.C.M of 72, 108 and 2100.

$72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$

$L.C.M = 2^3 \times 3^3 \times 5^2 \times 7 = 37800.$

Time and work.

1. Worker A takes 8 hours to do a job. worker B takes 10 hours to do same. How long should it take both A and B, working together but independently to do same job?.

A's 1 hour's work = $1/8$, B's 1 hour work = $1/10$.

(A+B)'s 1 hour's work = $(1/8 + 1/10) = 9/40$.

Both A and B will finish the work in $\frac{40}{9} = 4\frac{4}{9}$ days.

2). A and B together can complete a piece of work 4 days. If A alone can complete the same work 12 days. in many days can B alone complete that work?.

(A+B)'s 1 day's work = $1/4$ A's 1 day's work = $1/12$.

B's 1 day's work = $(1/4 - 1/12) = 1/6$.

Hence B alone can complete the work in 6 days.

3). A is twice good a workmann as B and together they finish a piece in 18 days how many days A alone finish the work?.

(A + B)'s 1 day's work = $1/18$.

Divide $1/18$ in ratio 2:1

A's 1 day's work = $(1/18 \times 2/3) = 1/27$.

4). A can do certain job 12 days. B in 60 y. more efficient than A how days does B alone take do the same job.

Ratio of time A and B = 160 : 100 = 8 : 5.

$8 : 5 :: 12 : x \Rightarrow 8x = 5 \times 12 \Rightarrow x = 7\frac{1}{2}$ days.

Problems on Ages:

1). Rajeev's age after 15 years will be 5 times his age 5 years back present age of Rajeev:

Let Rajeev's present age be x years Then.

Rajeev's age after 15 years = (x+15) years.

Rajeev's age 5 years back = (x - 5) years.

$x+15 = 5(x-5) \Rightarrow x+15 = 5x-25 \Rightarrow 4x = 40 \Rightarrow x = 10$.

= 10 years.

2). The ages of two persons differ by 16 years. If 6 years ago, the elder one was 3 times as old as younger one find their present ages.

Let the age of younger person be x years.

Then age of elder person = (x+16) years.

$3(x-6) = (x+16-6) \Rightarrow 3x-18 = x+10 \Rightarrow 2x = 28 \Rightarrow x = 14$.

3). A train 100 m long is running at speed 30 km/hr find the time taken by a man standing the railway line.

Speed of the train = $(30 \times 5/18)$ m/sec = $(\frac{25}{3})$ m/sec.

Distance moved in passing the standing man = 100m.

Required time taken = $\frac{100}{(25/3)} = (100 \times \frac{3}{25})$ sec = 12 sec.

4). A man sitting a train is travelling 50 kmph observes that a goods train travelling in opposite direction takes 9 seconds to pass him. If goods train 280 m long find its speed.

Relative speed = $(\frac{280}{9})$ m/sec = $(\frac{280}{9} \times \frac{18}{5})$ kmph = 112 kmph.

speed of goods train = (112 - 50) kmph = 62 kmph

Volume and surface Area:-

1). Find the volume and surface area of cuboid 16m long, 14m bread 7m high:-

Volume $= (16 \times 4 \times 7)m = 1568\ m^2$.

Surface area $= [2(16\times14 + 4\times7 + 16\times7)cm^2 - (2\times434)cm^2$
$= 868\ cm^2$.

2). Find the length of longest pole that can be placed room 12 m long 3m broad and 9m high.

Length of longest pole = length of diagonal of room.

$= \sqrt{(12)^2 + 8^2 + 9^2} = \sqrt{289} = 17m$.

3). The surface area of cube is 1734 sq.cm find its volume.

$6a^2 = 1734 \Rightarrow a^2 = 289 \Rightarrow a = 17cm$.

Volume $= a^3 = (17)^3 cm^3 = 4913\ cm^3$.

4). A cube of edge 15cm is immersed completely in rectangular vessel containing. If dimensions are 20cm and 15cm find the possible number of cubes.

Increase in volume = volume of the cube $= 15\times15\times15\ cm^3$.

Rise in water level $=$ $\frac{\text{volume}}{\text{Area}} = \left(\frac{15\times15\times15}{20\times15}\right)cm = 11.25cm$.

5). Three solid cubes of sides 1cm, 6cm and 8cm are melted to a new find the surface area of cube so formed.

volume of new cube $= (1^3 + 6^3 + 8^3)cm^3 = 729\ cm^3$.

Edge of new cube $= \sqrt[3]{729}\ cm = 9cm$.

Surface area of new cube $= (6\times9\times9)\ cm^2 = 486\ cm^2$.

Area:-

1). One side of a rectangular field is 15 m and one of diagonals 17 m. Find the area of field.

Other side $= \sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8$ m.

Area $= (15 \times 8)\ m^2 = 120\ m^2$.

2). A lawn in form of rectangle having its sides in ratio 2:3 the area of lawn 1/6 hectares. Find the length and breadth of lawn.

Let length $= 2x$ metres and breadth $= 3x$ metres.

$$\text{area} = \left(\tfrac{1}{6} \times 1000\right) m^2 = \left(\frac{5000}{3}\right) m^2.$$

$$2x \times 3x = \frac{5000}{3} \Rightarrow x^2 = \frac{2500}{9} \Rightarrow x = \left(\frac{50}{3}\right).$$

Length $= 2x = \frac{100}{3}$ m $= 33\frac{1}{3}$ m and Breadth $= 3x$

$= \left(3 \times \frac{50}{3}\right) m =$ 50 m.

3). Find the cost of carpeting a room 13 m long and 9 m broad with a 75 cm wide at rate Rs. 12.40 per square metre.

Area of carpet = Area of room $= (13 \times 9)\ m^2 = 117\ m^2$.

Length of the carpet $= \left(\frac{\text{Area}}{\text{width}}\right) = (117 \times 4/3)\ m = 156$ m.

Cost of carpeting $=$ Rs. $(156 \times 1240) =$ Rs. 1934.40.