

Equation (4) represents the Wien's law of distribution of energy.

### Rayleigh - Jean's law. (জ্বর জোন নিয়ম)

The energy distribution in the thermal spectrum according to Rayleigh, is given by the formula,

$$F_\lambda = \frac{8\pi kT}{c^3} \lambda^{-5}$$

Here,  $k$  is the Boltzmann's constant,

The experimental results of Lummer and Pringsheim in the infra red region, however show that the Wien's law holds good only in the region of shorter wavelengths.

It does not hold good at longer wavelengths.

• Thus Rayleigh - Jean's law holds good in the region of longer wavelengths but not for shorter wavelengths.

- This was shown by Rubens and Kuhnbaum.
- Thus Wien's law and Rayleigh - Jean's law do not precisely agree with the experimental results.

### Planck's Law (প্লানকেস নিয়ম)

• Planck (1901) was able to derive a theoretical expression for the energy distribution on the basis of quantum theory of heat radiations.

According to quantum theory, energy is emitted in the form of packets (or) quanta called photons.

- Each photon has an energy  $h\nu$  where  $h$  is the planck's Constant and  $\nu$  is the frequency of radiation.
  - According to this theory, the body does not emit energy continuously but only in certain multiples of the fundamental frequency of the resonator (energy emitter).
  - As the energy of a photon is  $h\nu$ , the energy emitted is equal to  $h\nu, 2h\nu, 3h\nu, \dots$ . He deduced the formula,
- $$E_\lambda = \frac{8\pi hcW}{c^5 \lambda^5 e^{hc/\lambda kT} - 1} \quad (6)$$
- Here,  $c$  is the velocity of electromagnetic waves ( $= 3 \times 10^8$  metres per Second). Equation (6) agrees with the experimental results.

(\*) A black body radiation chamber is not only filled up with the radiations but also with a large number of tiny oscillators. They are of atomic dimensions. Hence they are known as atomic oscillators.

Let us consider a black body consists a large number of atomic oscillators.

Average energy  $\bar{E}$  per oscillator is given by

$$\bar{E} = \frac{E}{N} \quad \text{--- (1)}$$

where,

$E \rightarrow$  Total energy of all oscillator

$N \rightarrow$  Number of oscillator

Number of atomic oscillators in the ground state = ~~No.~~  $N$ .

According to Maxwell energy distribution law the number of oscillator having energy  $E_n$  is given by,

$$N_n = N_0 e^{-E_n/kT} \quad \text{--- (2)}$$

where, T is absolute temperature of the black body.

k is Boltzmann's Const.

Here,  $N_1, N_2, N_3$  are the number of oscillators

N is the total number of oscillators

$E_0, E_1, E_2$  is the energies of oscillators

Then,

$$N = N_0 + N_1 + N_2 + \dots$$

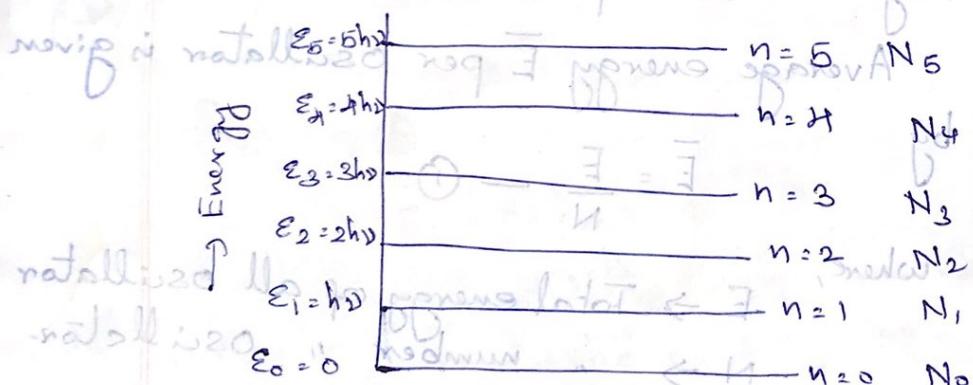
By using the eq. (2)

$$N = N_0 e^{-E_0/kT} + N_0 e^{-E_1/kT} + N_0 e^{-E_2/kT} \quad \text{--- (3)}$$

From Planck's quantum theory,  $E$  can take only a quant of value of  $h\nu$ .

So the possible values of  $E$  are  $0, 1, 2, 3, 4, 5, \dots$  etc.

$1, 2, 3, 4, 5, \dots$  etc. are called as quantum numbers.



$$\therefore E_n = nh\nu, \quad n=0, 1, 2, \dots$$

$$E_0 = 0, \quad E_1 = h\nu, \quad E_2 = 2h\nu, \quad E_3 = 3h\nu, \quad E_4 = 4h\nu, \quad E_5 = 5h\nu$$

Substituting these values in eq. (3), we have

$N = N_0 + N_1 + N_2 + N_3 + N_4 + N_5$

$$N = N_0 e^0 + N_0 e^{-h\nu/kT} + N_0 e^{-2h\nu/kT} + \dots$$

$$N = N_0 + N_0 e^{-h\nu/kT} + N_0 e^{-2h\nu/kT} + \dots \quad (4)$$

Put  $\alpha = e^{-h\nu/kT}$ , we have.  $[\because e^0 = 1]$

$$N = N_0 + N_0 \alpha + N_0 \alpha^2 + N_0 \alpha^3 + \dots \quad (5)$$

Total number of oscillators:

$$N = N_0 [1 + \alpha + \alpha^2 + \dots] \quad (6)$$

$$\therefore \frac{1}{1-\alpha} = (1-\alpha)^{-1} = 1 + \alpha + \alpha^2 + \dots$$

by using binomial Series.

$$\text{Total energy } E = \varepsilon_0 N_0 + \varepsilon_1 N_1 + \varepsilon_2 N_2 + \dots \quad (7)$$

Substituting the values for  $\varepsilon_0, \varepsilon_1, \varepsilon_2$ , and  $N_0, N_1, N_2 \dots$  in eq (7), we have.

$$E = \alpha N_0 + h\nu N_1 + 2h\nu N_2$$

$$E = 0 \times N_0 + h\nu N_0 e^{-h\nu/kT} + 2h\nu N_0 e^{-2h\nu/kT} + \dots$$

$$E = h\nu N_0 e^{-h\nu/kT} + 2h\nu N_0 e^{-2h\nu/kT} + \dots$$

Put  $\alpha = e^{-h\nu/kT}$ , we have.  $(8)$

~~similarly find~~  $E = h\nu N_0 \alpha + 2h\nu N_0 \alpha^2 + \dots \quad (9)$

~~both bnd~~  $E = h\nu N_0 [\alpha + 2\alpha^2 + \dots]$

$$E = h\nu N_0 \alpha [1 + 2\alpha + \dots] \quad (B)$$

~~EE~~  $E = h\nu N_0 \alpha \left[ \frac{1}{(1-\alpha)^2} \right] \quad (10)$

Total energy of the oscillators:

$$\therefore \frac{1}{(1-\alpha)^2} = (1-\alpha)^{-2} = 1 + 2\alpha + \dots$$

by using binomial Series.

Substituting the eq ⑥ and ⑩ in eq ① we get

$$\bar{E} = \frac{h\nu N_0 x}{(1-x)^2}$$

$$⑪ \quad \bar{E} = \frac{h\nu N_0 x}{(1-x)^2} \cdot \frac{N_0}{N_0 + N_1 + N_2 + \dots}$$

$$\bar{E} = \frac{h\nu N_0 x \cdot (1-x)}{(1-x)^2 \cdot N_0 + N_1 + N_2 + \dots}$$

$$= \frac{h\nu x}{x(1-x)}$$

$$[x + x + (x+1) - (1-x)] \cdot \frac{1}{x(1-x)}$$

$$\text{using binomial} \quad \bar{E} = \frac{h\nu x}{x(\frac{1}{x}-1)}$$

$$⑫ \quad \bar{E} = h\nu$$

$$\text{now, } e^{-h\nu/kT} \text{ is constant at } \frac{1}{x}$$

On Substituting,  $x = e^{-h\nu/kT}$ , we have

$$\bar{E} = \frac{h\nu}{e^{-h\nu/kT} + e^{-h\nu/kT} + \dots}$$

$$⑬ \quad \bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1}$$

Number of oscillators per unit volume in the wavelength range  $\lambda$  and  $\lambda + d\lambda$  is given by

$$n = \frac{8\pi d\lambda}{\lambda^4}$$

The energy density of radiation between wavelengths  $\lambda$  and  $\lambda + d\lambda$  is given by

$$E_\lambda d\lambda = \left[ \begin{array}{l} \text{Number of oscillators} \\ \text{per unit volume} \\ \text{in the interval} \\ \lambda \text{ and } \lambda + d\lambda \end{array} \right] \times \begin{array}{l} \text{Average} \\ \text{energy} \\ \text{per} \\ \text{oscillator} \end{array}$$

$$E_\lambda d\lambda = n \bar{E}$$

$$E_\lambda d\lambda = \frac{8\pi d\lambda}{\lambda^4} \times \frac{h\omega}{e^{h\omega/kT} - 1} \quad \text{--- (14)}$$

$$= \frac{8\pi d\lambda}{\lambda^4} \times \frac{hc/\lambda}{e^{h\omega/kT} - 1} \quad \left[ \because \omega = \frac{c}{\lambda} \right]$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{h\omega/kT} - 1} d\lambda \quad \text{--- (15)}$$

$$\boxed{E_\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{h\omega/kT} - 1}} \quad \text{--- (16)}$$

This eq (16) represents Planck's radiation law in terms of wavelength

— X —

① For shorter wavelengths

from eq (6),  $E_\lambda = \frac{8\pi h c}{\lambda^5} e^{-\frac{hc}{kT}}$

$$E_\lambda = \frac{8\pi h c}{\lambda^5} e^{-\frac{hc}{kT}}$$

or since at short wavelengths  $\frac{1}{\lambda^5}$  is dominant, —

$$E_\lambda \approx C_1 \lambda^{-5} e^{-\frac{hc}{kT}}$$

where,  $C_1 = 8\pi h c$  and  $C_2 = \frac{hc}{kT}$

Equation ① represents Wien's radiation law.

For longer wavelengths,  $\frac{1}{\lambda^5}$  is small.

From eq (6), expanding  $e^{-\frac{hc}{kT}}$  and neglecting higher power terms, we get

$$\textcircled{2} \rightarrow E_{\lambda} = \frac{8\pi h c}{\lambda^5 \left[ 1 + \frac{h\nu}{kT} - 1 \right]} = \frac{8\pi h c \cdot kT}{\lambda^5 h\nu} = \frac{8\pi c k T}{\lambda^3}$$

Cancelling out  $\frac{8\pi h c}{kT}$

~~$$E_{\lambda} = \frac{8\pi h c \cdot k T}{\lambda^4 \cdot h c}$$~~

~~$$E_{\lambda} = \frac{8\pi k T}{\lambda^4}$$~~

70. red mark  $E_{\lambda} = \frac{8\pi k T}{\lambda^4}$   $\rightarrow$   $\textcircled{8}$

Equation  $\textcircled{8}$  represents Rayleigh-Jean's law  
thus Planck's formula for the energy distribution  
in a thermal spectrum is applicable for all  
wavelengths.

## Solar Constant (सौर स्टॉन्ट)

the Sun is the source of heat radiation, and it emits heat radiations in all directions. The earth receives only a fraction of the heat energy emitted by the Sun.

The atmosphere also absorbs a part of the heat radiations and air, clouds, dust particles etc. in the atmosphere scatter the heat and light radiations falling on them.

From the quantity of heat radiations received by the earth it is possible to estimate the temperature of the Sun.

Therefore, to determine the value of a constant, called Solar Constant, certain ideal conditions are taken into consideration.

It is the amount of heat energy (radiation) absorbed per minute by one  $89\text{ cm}^2$  of a perfectly black body surface placed at a mean distance of the earth from the Sun, in the absence of the atmosphere, the surface being held perpendicular to the Sun's rays.

The instrument used to measure the Solar Constant are called pyrheliometers.

## Angstrom's Pyrheliometer (अंगस्ट्रॉम वायरिमीटर)

A pyrheliometer is an instrument which is used to find the amount of incident heat radiation and the Solar Constant.

Angstrom's pyrheliometer consists of two strips (identical) strips A and B of blackened platinum foil.

A strip A is exposed to the Sun and is shielded by a cover C.

A thermocouple having a sensitive galvanometer with A as one junction and B as the other junction is used.

The strip B can be heated by an electrical arrangement and suitable current passing through B can be adjusted with the help of a rheostat.

When both the strips A and B are shielded from the Sun their junctions are at the same temperature and the galvanometer shows no deflection in the galvanometer.

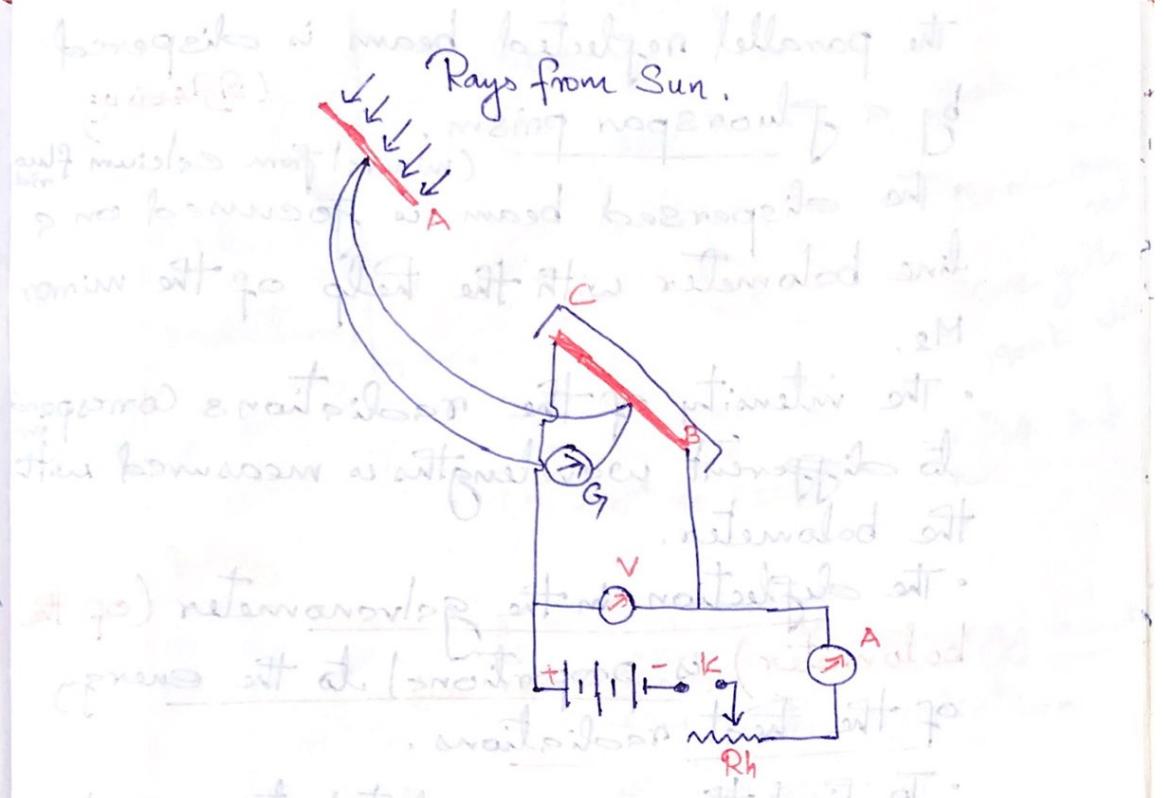
It means the strip A and B are again at the same temperature and they are receiving heat energy at the same rate.

Let H calories of heat be incident on one sq. cm surface of the strip in one minute.

The area of the plate A = A sq. cm and absorption co-efficient =  $\alpha$

The amount of heat radiations absorbed in one minute by the plate A =  $H A \alpha$  calories

Heat produced in one minute in the strip B,  $\frac{I \times 60}{4.2}$  cal/cm<sup>2</sup>



Here  $E$  volts is the potential difference across the strip  $B$  and  $I$  amperes is the current flowing through it.

Equating (1) and (2)

$$H A \propto E I \times 60 \quad \text{--- (3)}$$

Hence  $H$  can be calculated.

**Distribution of energy in the Spectrum of a Black body.** (புதினங்களில் வெள்ளும் ஊத்தி)

- Lummer and Pringsheim investigated the distribution of energy amongst the different wavelengths of a thermal spectrum of a black body radiation (full emitter).
- The experimental arrangement consists of a carbon tube heated electrically.
- The radiations from this tube are allowed to be incident on a reflector  $M$ , through a slit  $S_1$ .