Electrical Measurements

7.1. Carey Foster Bridge

Description. The Carey Foster bridge is a form of Wheatstone's bridge. It consists of a uniform wire *AB* of length 1 metre stretched on a wooden board (Fig. 7.1).

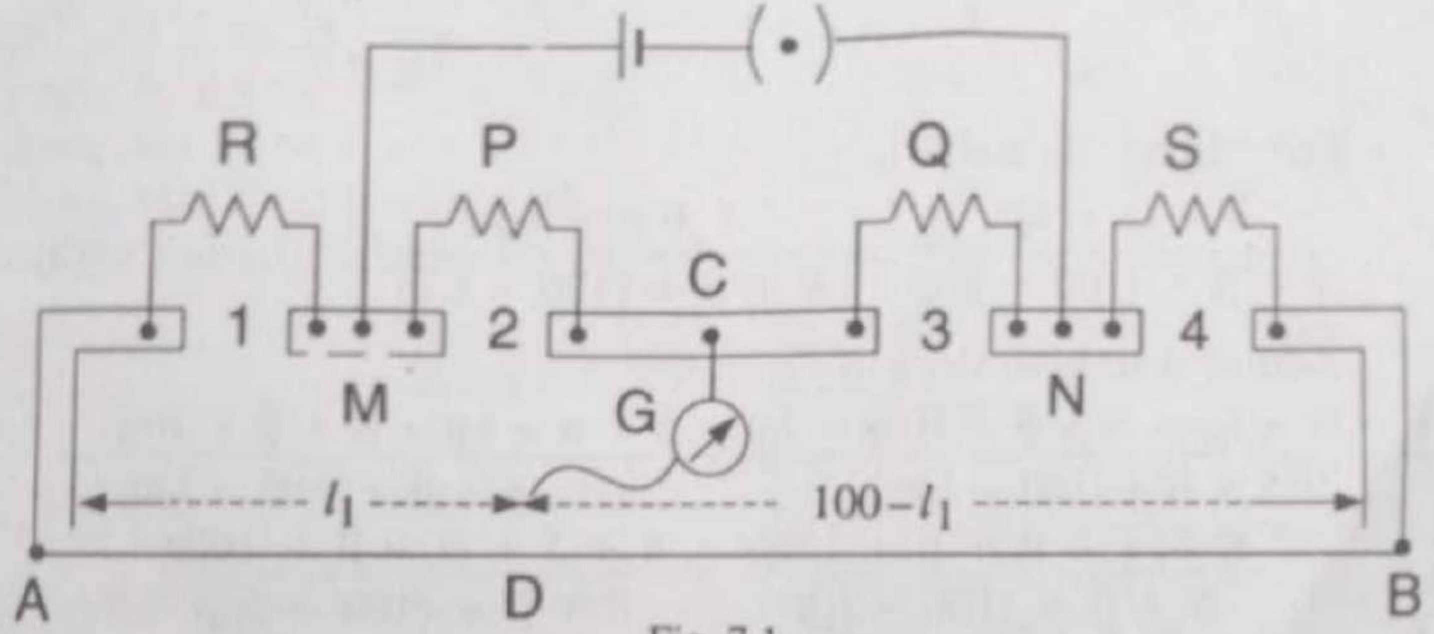

Fig. 7.1

Two equal resistances *P* and *Q* are connected in gaps 2 and 3. The unknown resistance *R* is connected in gap 1. A standard resistance *S*, of the same order of resistance as *R*, is connected in gap 4. A Leclanche cell is connected across *MN*. A galvanometer *G* is connected between the terminal *C* and a sliding contact maker *D*.

Theory. The contact maker is moved until the bridge is balanced. Let l_1 be the balancing length as measured from end *A*. Let α and β be the end resistances at *A* and *B*. Let ρ be the resistance per unit length of the wire.

From the principle of Wheatstone's bridge,

$$\frac{P}{Q} = \frac{R + \alpha + l_1\rho}{S + \beta + (100 - l_1)\rho} \qquad ...(1)$$

The resistances *R* and *S* are interchanged and the bridge is again balanced. The balancing length l_2 is determined from the same end *A*. Then,

$$\frac{P}{Q} = \frac{S + \alpha + l_2\rho}{R + \beta + (100 - l_2)\rho} \quad ...(2)$$

Figs. 7.2 and 7.3 represent the equivalent Wheatstone's bridge circuit in the two cases.

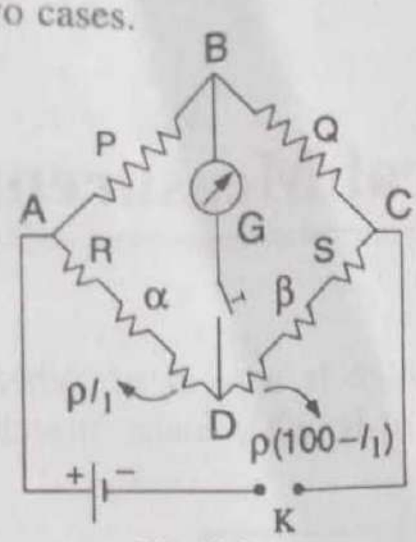

Fig. 7.2

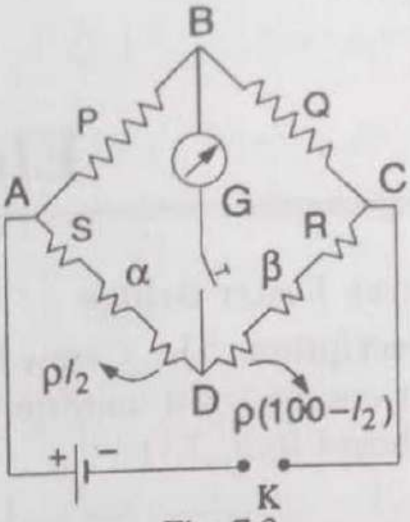

Fig. 7.3

From Eqns. (1) and (2),

$$\frac{R + \alpha + l_1\rho}{S + \beta + (100 - l_1)\rho} = \frac{S + \alpha + l_2\rho}{R + \beta + (100 - l_2)\rho} \quad ...(3)$$

Adding 1 to both sides of Eq. (3),

$$\frac{R + \alpha + l_1\rho + S + \beta + 100\rho - l_1\rho}{S + \beta + (100 - l_1)\rho} = \frac{S + \alpha + l_2\rho + R + \beta + 100\rho - l_2\rho}{R + \beta + (100 - l_2)\rho}$$

$$\therefore \quad \frac{R + S + \alpha + \beta + 100\rho}{S + \beta + (100 - l_1)\rho} = \frac{R + S + \alpha + \beta + 100\rho}{R + \beta + (100 - l_2)\rho}$$

Since the numerators are equal, the denominators must be equal.

$$\therefore \quad S + \beta + 100\rho - l_1\rho = R + \beta + 100\rho - l_2\rho \quad ...(4)$$

or

$$S - l_1\rho = R - l_2\rho$$

$$\therefore \quad R = S + \rho\,(l_2 - l_1) \quad ...(5)$$

To find ρ. A standard resistance of 0.1 Ω is connected in gap 1. A thick copper strip is connected in gap 4 *i.e.*, $R = 0.1\,\Omega$ and $S = 0$. The balancing length l_1' is determined. The standard resistance and the thick copper strip are interchanged. The balancing length l_2' is determined.

From Eq. (5),
$$0.1 = S + \rho\,(l_2' - l_1')$$

or
$$\rho = \frac{0.1}{(l_2' - l_1')}$$

Thus by knowing S and ρ, the unknown resistance R is calculated.

Determination of the temperature coefficient of resistance

Let R_0 and R_t be the resistances of a wire at temperatures 0°C and t°C. Then,

$$R_t = R_0(1 + \alpha t)$$

or
$$\alpha = \frac{R_t - R_0}{R_0 t} = \frac{1}{R_0}\frac{dR}{dt}$$

where α is the temperature coefficient of resistance of the material.

The increase of resistance per unit original resistance per degree rise of temperature is called temperature coefficient of resistance.

The given wire is wound non-inductively in the form of a double spiral on a glass tube. It is immersed in a beaker containing ice at 0°C. The resistance of the wire is determined as above. The resistance of the wire is determined at 10°, 20°, 30°, ... 100°C. A graph is drawn with temperature along the X-axis and resistance along the Y-axis (Fig. 7.4). A straight line is obtained.

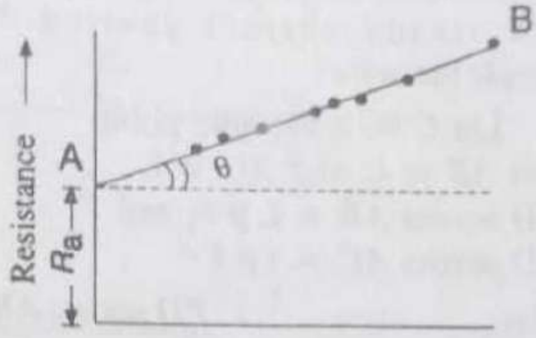

Fig. 7.4

$$\left.\begin{matrix}\text{Slope of}\\ \text{the line}\end{matrix}\right\} = \tan\theta = \frac{dR}{dt}.$$

$$Y \text{ intercept} = R_0.$$

α is calculated using the formula, $\alpha = \dfrac{1}{R_0}\dfrac{dR}{dt}$

Note. Let R_1 and R_2 be the resistances at t_1^0C and t_2^0C respectively.

$$R_1 = R_0[1 + \alpha t_1] \text{ and } R_2 = R_0[1 + \alpha t_2]$$

$$\therefore \qquad \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

Example 1. *In an experiment with Carey Foster bridge, the shift in the balance point is 5.4 cm when a thick copper strip and one ohm resistance are interchanged. The one ohm resistance is then replaced by an unknown resistance. Now the balance point shifts by 10 cm on interchanging. Calculate the unknown resistance.*

Sol.
$$\rho = \frac{1}{5.4} \text{ ohm/cm}$$

$$R = S + \rho(l_2 - l_1) = 0 + \frac{1}{5.4} \times 10 = 1.85 \text{ ohm.}$$

7.2. Potentiometer

Principle. A potentiometer is a device for measuring or comparing potential differences. A potentiometer can be used to measure any electrical quantity which can be converted into a proportionate D.C. potential difference.

It consists of a uniform wire AB of length 10 m stretched on a wooden board (Fig. 7.5). A steady current is passed through the wire AB with the help of a cell of EMF E. Let

ρ = resistance per unit length of potentiometer wire, and

I = steady current passing through the wire.

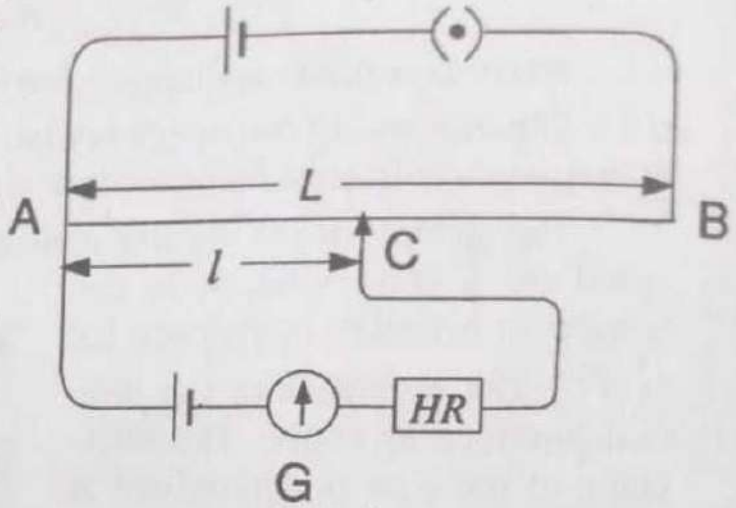

Fig. 7.5

Let C be a variable point.

Let $AB = L$ and $AC = l$.

PD across $AB = L\rho I$, and

PD across $AC = l\rho I$

$$\frac{PD \text{ across } AB}{PD \text{ across } AC} = \frac{L\rho I}{l\rho I} = \frac{L}{l}$$

$$\therefore \quad PD \text{ across } AC = \frac{l}{L} \times PD \text{ across} AB$$

i.e., for a steady current passing through the potentiometer wire AB, the PD across any length is proportional to the length of the wire.

If a D.C. voltmeter is connected between A and the variable point C, it will be noted that the voltmeter registers greater values of PD's as the point C slides from A to B.

Comparison of EMFs of two cells

A lead accumulator of emf E is connected across the potentiometer wire AB (Fig. 7.6). G is a galvanometer. E_1 and E_2 are the two emf's to be compared. The emf of E_1 is balanced across a length l_1 of the potentiometer. Similarly, the emf of E_2 is balanced across a length l_2 of the potentiometer.

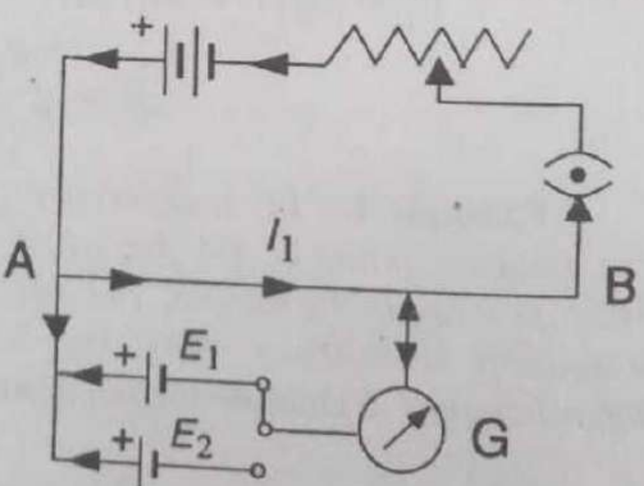

Fig. 7.6

Then,

$$E_1 \propto l_1 \text{ and } E_2 \propto l_2$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{l_1}{l_2}$$

If one of the cells, say E_1, is a standard cadmium cell, the emf of the other can be determined.

Determination of the Internal Resistance of a Cell

AB is the potentiometer wire (Fig. 7.7). A steady current is passed through the wire with the help of a battery. E is the cell whose internal resistance is to be measured. A resistance box R is connected across the cell through a key K_2.

Closing key K_1, a balancing point is obtained on the potentiometer wire with K_2 open. The balancing length l_1 (= AC) now is a measure of the EMF E of the cell. Then,

$$E \propto l_1 \quad \text{...(1)}$$

The key K_2 is closed. A resistance R is introduced in the box. Without disturbing rheostat Rh, the balancing length l_2 (= AD) is measured. This is a measure of the PD V of the cell. Then

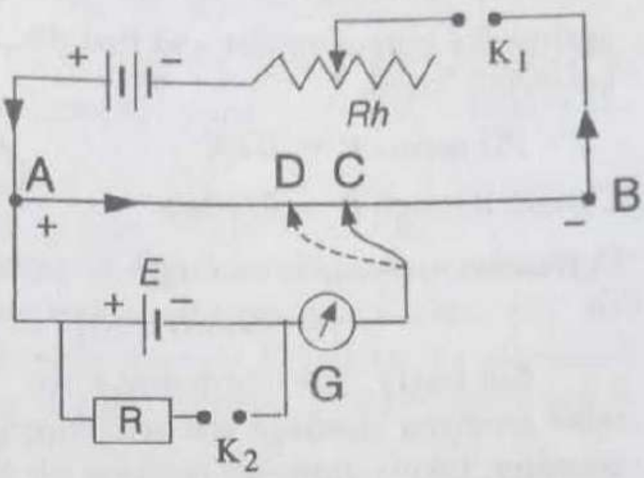

Fig. 7.7

$$V \propto l_2 \quad \text{...(2)}$$

From Eqns. (1) and (2), $\dfrac{E}{V} = \dfrac{l_1}{l_2}$ ...(3)

Let E be the EMF of the cell and r the internal resistance. Let V be the PD across the cell when supplying a current I through the external resistance R.

Then,

$$V = IR \quad \text{...(4)}$$

and

$$E = I(R + r) \quad \text{...(5)}$$

$$\therefore \quad \frac{E}{V} = \frac{R + r}{R} = 1 + \frac{r}{R} \quad \text{...(6)}$$

Comparing Eqns. (3) and (6), $1 + \dfrac{r}{R} = \dfrac{l_1}{l_2}$

$$\therefore \quad r = \left(\frac{l_1 - l_2}{l_2}\right)R \quad \text{...(7)}$$

Hence r is calculated.

Calibration of Ammeter

Connect the ends of the potentiometer wire to the terminals of a storage cell through a key K_1 (Fig. 7.8). S is a standard cell. Connect the ammeter (A) to be calibrated in series with a battery, key K_2, a rheostat and a standard resistance R. When a current I passes through the standard resistance R, the PD across R is IR. This potential drop is measured with the help of potentiometer.

Connect 1 and 3 and balance the EMF of the standard cell against the potentiometer. Find the balancing length from A. The PD per cm of the potentiometer = E/l.

Connect 2 and 3. Adjust the rheostat so that the ammeter reads a value A_1. Balance the PD across R against the potentiometer and find the balancing length l_1.

PD across $R = E l_1/l$

Current through $R = El_1/(lR)$.

Correction to ammeter reading

$$= (El_1/lR) - A_1$$

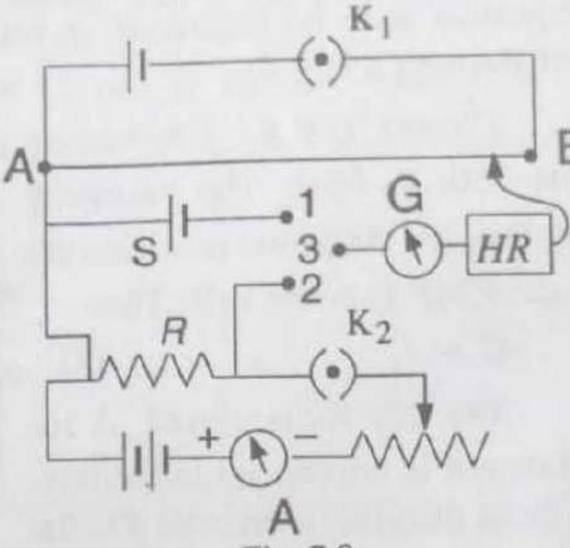

Fig. 7.8

Similarly, the corrections for other ammeter readings are determined. A calibration curve is plotted for ammeter, taking ammeter readings on X-axis and corrections on Y- axis.

Calibration of voltmeter (Low range)

The connections are made as shown in Fig. 7.9. The voltmeter is connected parallel to R. Let l be the balancing length for the standard cell. The PD across R is balanced against the potentiometer. Let l_1 be the balancing length when the voltmeter reads V_1.

PD across $R = El_1/l$

Correction to voltmeter

$$= (El_1/l) - V_1$$

The experiment is repeated for various readings of the voltmeter and a calibration graph is drawn.

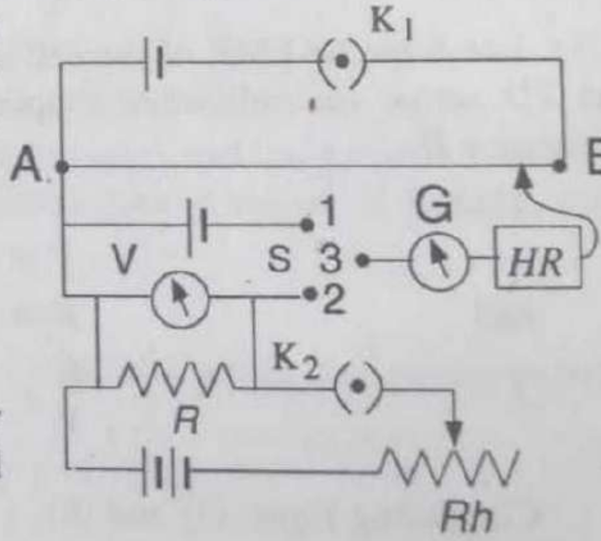

Fig. 7.9

Calibration of voltmeter (High Range)

Connections are made as shown in Fig. 7.10. Take suitable high resistances in P and Q such that the PD across P does not exceed the PD across the potentiometer. The balancing length l for the standard cell is determined first. Then the PD across P is balanced against the potentiometer and the balancing length l_1 is determined.

PD across $P = El_1/l$

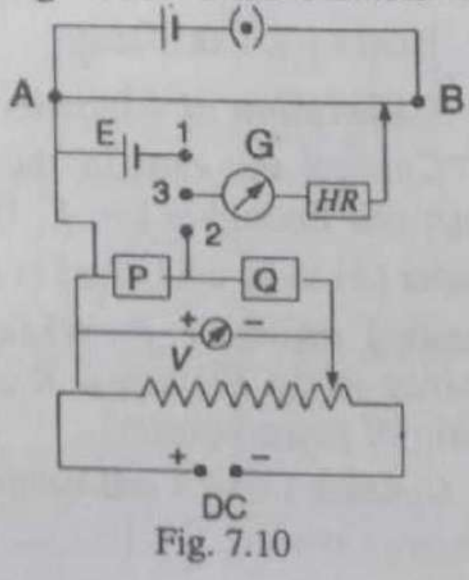

Fig. 7.10

$$\text{PD across } P + Q = \left(\frac{P+Q}{P}\right)\left(\frac{El_1}{l}\right)$$

$$\text{Correction to voltmeter} = \left(\frac{P+Q}{P}\right)\left(\frac{El_1}{l}\right) - V_1.$$

The experiment is repeated for various readings of the voltmeter. A calibration curve is plotted for voltmeter, taking voltmeter readings on X-axis and corrections on Y-axis.

Comparison of Resistances

Connections are made as shown in Fig. 7.11. The two resistances P and Q to be compared are connected in series with a battery, key and rheostat. The same current I is flowing through P and Q. To determine potential drop across P, connection is made between 1 and 4, and 2 and 5. To determine potential across Q, connection is made between 2 and 4, and 3 and 5. The PD across P is first balanced against the potentiometer and the balancing length l_1 is determined. Without altering the current in P and Q, the PD across Q is balanced against the potentiometer and the balancing length l_2 is determined. Then,

$$IP \propto l_1 \text{ and } IQ \propto l_2$$

$$\therefore \quad \frac{P}{Q} = \frac{l_1}{l_2}$$

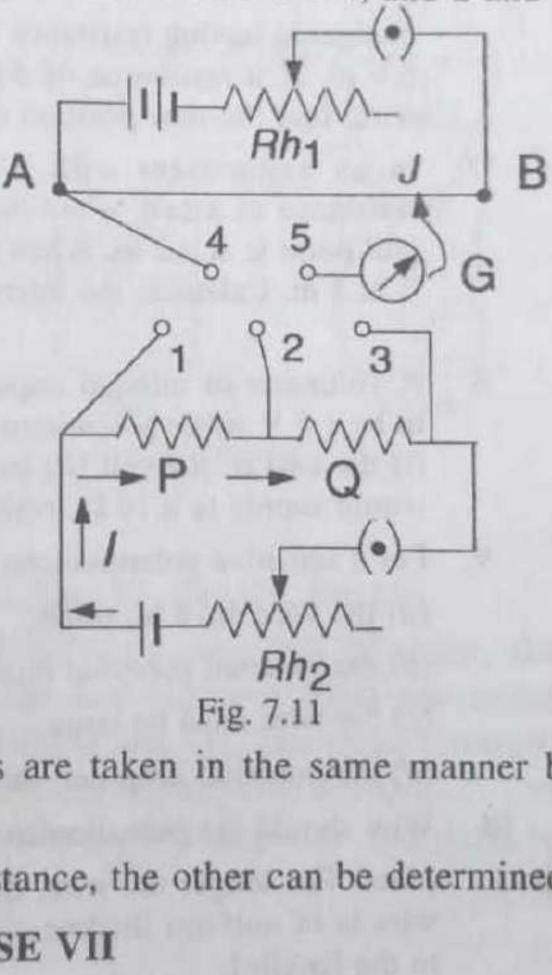

Fig. 7.11

At least six sets of observations are taken in the same manner by sliding the rheostat Rh_2.

If one of them is a standard resistance, the other can be determined.

EXERCISE VII

1. Explain with necessary theory how a Carey Foster bridge may be used to compare two nearly equal resistances. Hence show how the specific resistance of the material of the wire can be determined.
2. Define "temperature coefficient of resistance". How is it determined using the Carey-Foster bridge ?
3. Explain the theory of a potentiometer. How will you use it to :
   (*a*) compare the emf of two cells.
   (*b*) find the internal resistance of a cell

Thermo-electricity

8.1. Seebeck Effect.

When two dissimilar metal wires are joined together so as to form a closed circuit and if the two junctions are maintained at different temperatures, an emf is developed in the circuit (Fig. 8.1). This causes a current to flow in the circuit as indicated by the deflection in the galvanometer G. This phenomenon is called the *Seebeck effect.* This arrangement is called a *thermocouple*. The emf developed is called *thermo emf.* The thermo emf so developed depends on the temperature difference between the two junctions and the metals chosen for the couple. Seebeck arranged the metals in a series as follows :

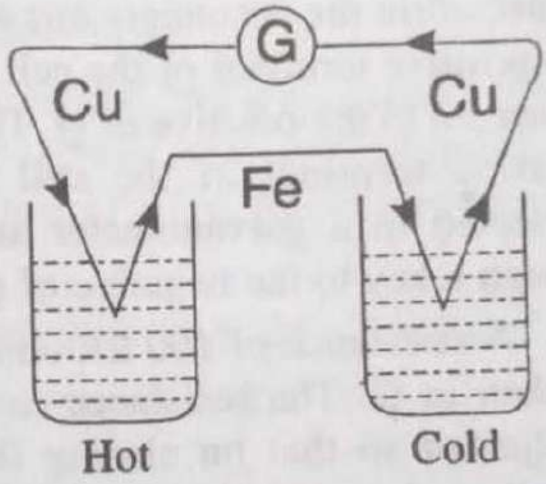

Fig. 8.1

Bi, Ni, Pd, Pt, Cu, Mn, Hg, Pb, Sn, Au, Ag, Zn, Cd, Fe, Sb.

When a thermocouple is formed between any two of them, the thermoelectric current flows through the hot junction from the metal occurring earlier to the metal occurring later in the list. The more removed are the two metals in the list, the greater is the thermo emf developed. The metals to the left of Pb are called *thermoelectrically negative* and those to its right are *thermoelectrically positive.*

8.2. Laws of thermo e.m.f.

(*i*) **Law of Intermediate Metals.** *The introduction of any additional metal into any thermoelectric circuit does not alter the thermo emf provided the metal introduced is entirely at the same temperature as the point at which the metal is introduced.*

If ${}_aE_b$ is the emf for a couple made of metals A and B, and ${}_bE_c$ that for the couple of metals B and C, then the emf for couple of metals A and C is given by

$${}_aE_c = {}_aE_b + {}_bE_c$$

(ii) **Law of Intermediate Temperatures.** *The thermo emf E_1^3 of a thermocouple whose junctions are maintained at temperatures T_1 and T_3 is equal to the sum of the emfs E_1^2 and E_2^3 when the junctions are maintained at temperatures T_1, T_2 and T_2, T_3 respectively.* Thus

$$E_1^3 = E_1^2 + E_2^3$$

8.3. Measurement of Thermo EMF using Potentiometer

Thermo emfs are very small, of the order of only a few millivolts. Such small emfs are measured using a potentiometer. A ten-wire potentiometer of resistance R is connected in series with an accumulator and resistance boxes P and Q (Fig. 8.2). A standard cell of emf E is connected in the secondary circuit. The positive terminal of the cell is connected to the positive of Q. The negative terminal of the cell is connected to a galvanometer and through a key to the negative of Q.

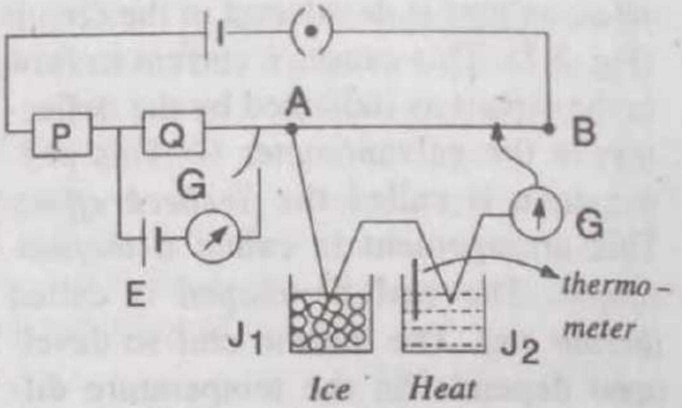

Fig. 8.2

A resistance of $100\ ER$ ohms is taken in Q. The resistance in P is adjusted so that on closing the key, there is no deflection in the galvanometer. Now, the PD across $100\ ER$ ohms is equal to E.

$$\left.\begin{array}{l}\text{PD across } R \text{ ohms}\\ \text{of the potentiometer}\end{array}\right\} = \frac{ER}{100\ ER}\ \text{volt} = \frac{1}{100}\ \text{volt} = 10\ \text{millivolt.}$$

Thus the fall of potential per metre of the potentiometer wire is 1 millivolt. So we can measure thermo emf up to 10 millivolt.

Without altering the resistances in P and Q, the positive of the thermocouple is connected to the positive terminal of the potentiometer and the negative of the thermocouple to a galvanometer and jockey. One junction is kept in melting ice and the other junction in an oil bath or in a sand bath. The jockey is moved till a balance is obtained against the small emf e of the thermocouple. Let $AJ = l$ cm be the balancing length. Then,

$$\text{thermo emf } e = \frac{1}{100}\ l\ \text{millivolt.}$$

Keeping the cold junction at 0°C, the hot junction is heated to different temperatures. The thermo emf generated is determined for different temperatures of the hot junction. A graph is drawn between thermo emf and the temperature of the hot junction (Fig. 8.3). The graph is a parabolic curve.

The thermo emf E varies with temperature according to

$E = at + bt^2$, where a and b are constants. The thermo emf increases as the temperature of the hot junction increases, reaches a maximum value T_n, then decreases to zero at a parti-cular temperature T_i. On further increasing the difference of temperature, emf is reversed in direction.

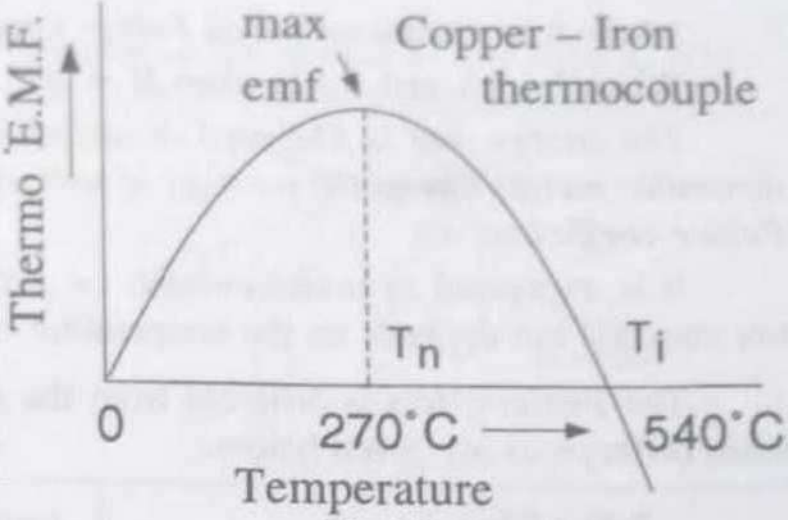

Fig. 8.3

For a given temperature of the cold junction, the temperature of the hot junction for which the thermo emf becomes maximum is called the neutral temperature (T_n) for the given thermocouple.

For a given temperature of the cold junction, the temperature of the hot junction for which the thermo emf becomes zero and changes its direction is called the inversion temperature (T_i) for the given thermocouple.

T_n is a constant for the pair of metals. T_i is variable. T_i is as much above the neutral temperature as the cold junction is below it.

8.4. Peltier Effect

Consider a copper-iron thermocouple (Fig. 8.4). When a current is allowed to pass through the thermocouple in the direction of arrows (from A to B), heat is absorbed at the junction B and generated at the junction A. This absorption or evolution of heat at a junction when a current is sent through a thermocouple is called Peltier effect. The Peltier effect is a reversible phenomenon. If the direction of the current is reversed, then there will be cooling at the junction A and heating at the junction B.

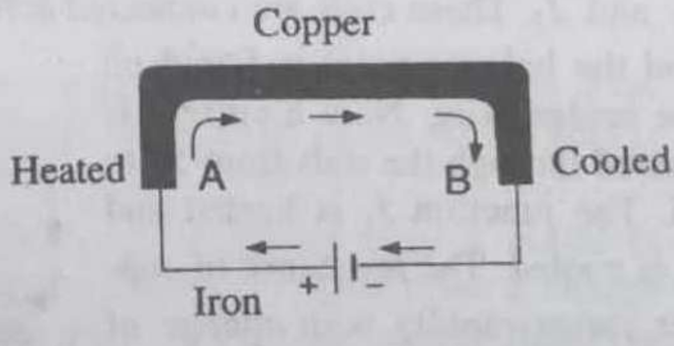

Fig. 8.4

When an electric current is passed through a closed circuit made up of two different metals, one junction is heated and the other junction is cooled. This is known as Peltier effect.

The amount of heat H absorbed or evolved at a junction is proportional to the charge q passing through the junction. *i.e.*,

$$H \propto q \quad \text{or} \quad H \propto It$$

or

$$H = \pi It$$

where π is a constant called *Peltier coefficient.*

When $I = 1A$ and $t = 1s$, then $H = \pi$.

The energy that is liberated or absorbed at a junction between two dissimilar metals due to the passage of unit quantity of electricity is called Peltier coefficient.

It is expressed in joule/coulomb *i.e.*, volt. The Peltier coefficient is not constant but depends on the temperature of the junction.

The Peltier effect is different from the I^2R Joule heating effect. The main differences are given below.

	Peltier Effect	*Joule Effect*
1.	It is a reversible effect.	It is an irreversible effect.
2.	It takes place at the junctions only.	It is observed throughout the conductor.
3.	It may be a heating or a cooling effect.	It is always a heating effect.
4.	Peltier effect is directly proportional to I ($H = \pm \pi It$)	Amount of heat evolved is directly proportional to the square of the current.
5.	It depends upon the direction of the current.	It is independent of the direction of the current.

Demonstration of Peltier effect – S.G. Starling Method.

Fig. 8.5 shows a bismuth bar between two bars of antimony. Two coils C_1 and C_2 of insulated copper wire are wound over the two junctions J_1 and J_2. These coils are connected across the two gaps of a metre-bridge and the balance-point is found on the bridge wire. Now a current is passed through the rods from *Sb* to *Bi*. The junction J_1 is heated and J_2 is cooled. The resistance of copper varies rapidly with change of temperature. Hence the balance in the bridge is immediately upset. The galvanometer shows a deflection. If the current is reversed, the deflection in the galvanometer also gets reversed. This shows that the junction J_1 is now cooled and J_2 is heated.

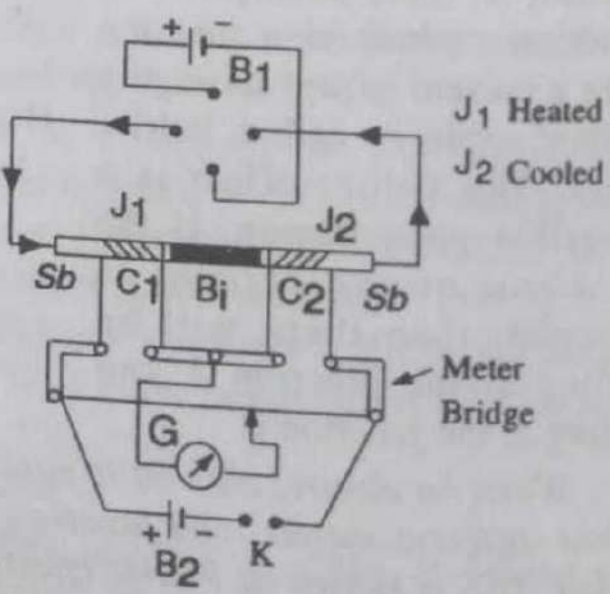

Fig. 8.5

8.5. Thomson Effect

Consider a copper bar AB heated in the middle at the point C (Fig. 8.6). A current is passed from A to B. It is observed that heat is absorbed

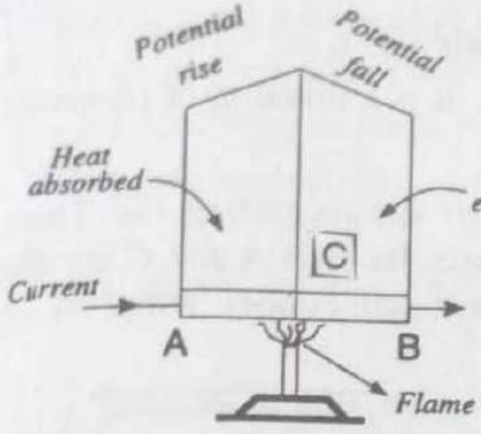

Fig. 8.6

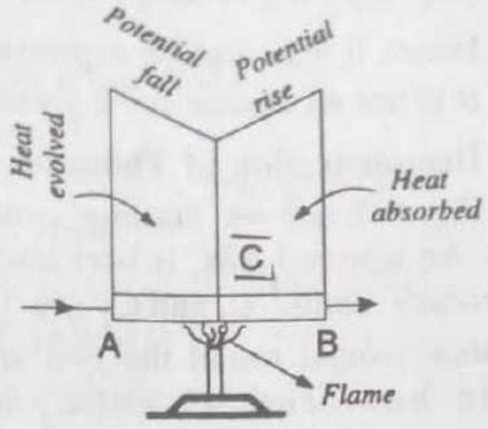

Fig. 8.7

in the part AC and evolved in the part CB. This is known as *Positive Thomson effect*. Similar effect is observed in metals like Ag, Zn, Sb and Cd.

In the case of an iron bar AB, heat is evolved in the part AC and absorbed in the part CB (Fig. 8.7). This is known as *Negative Thomson effect*. Similar effect is observed in metals like Pt, Ni, Co and Bi.

For lead, the Thomson effect is zero.

The Thomson effect is reversible.

In the case of copper, the hotter parts are at a higher potential than the colder ones. It is opposite in the case of iron. Heat is either absorbed or evolved when current passes between two points having a difference of potential. Therefore, the passage of electric current through a metal having temperature gradient results in an absorption or evolution of heat in the body of the metal.

When a current flows through an unequally heated metal, there is an absorption or evolution of heat throughout in the body of the metal. This is known as 'Thomson effect'.

Thomson Coefficient. *The Thomson coefficient σ of a metal is defined as the amount of heat energy absorbed or evolved when a charge of 1 coulomb flows in the metal between two points which differ in temperature by* $1^{\circ}C$.

Thus, if a charge of q coulomb flows in a metal between two points having a temperature difference of 1°C, then

heat energy absorbed or evolved = σq joule.

But if E volt be the Thomson emf developed between these points then this energy must be equal to Eq joule.

$$\therefore \quad \sigma q = Eq$$

or

$$\sigma = E.$$

Thus *the Thomson coefficient of a metal, expressed in joule per coulomb per °C, is numerically equal to the emf in volt, developed between two points differing in temperature by 1°C.*

Hence it may also be expressed in volt per °C.

σ is not a constant for a given metal. It is a function of temperature.

Demonstration of Thomson effect.

Fig. 8.8. shows Starling's method of demonstrating the Thomson effect. An iron rod *ABC* is bent into *U* shape. Its ends *A* and *C* are dipped in mercury baths. C_1 and C_2 are two insulated copper wires of equal resistance wound round the two arms of the bent rod. C_1 and C_2 are connected in the opposite gaps of a metre bridge. The bridge is balanced. Then the mid-point *B* of the rod is strongly heated. A heavy current is passed through the rod. Then this current will be flowing up the temperature gradient in one arm and down the temperature gradient in the other arm. As a result, one of the coils will be cooled and the other will be warmed. The balance in the bridge will be upset and the galvanometer in its circuit will show a deflection. If the direction of the current is reversed, the deflection in the galvanometer will be reversed.

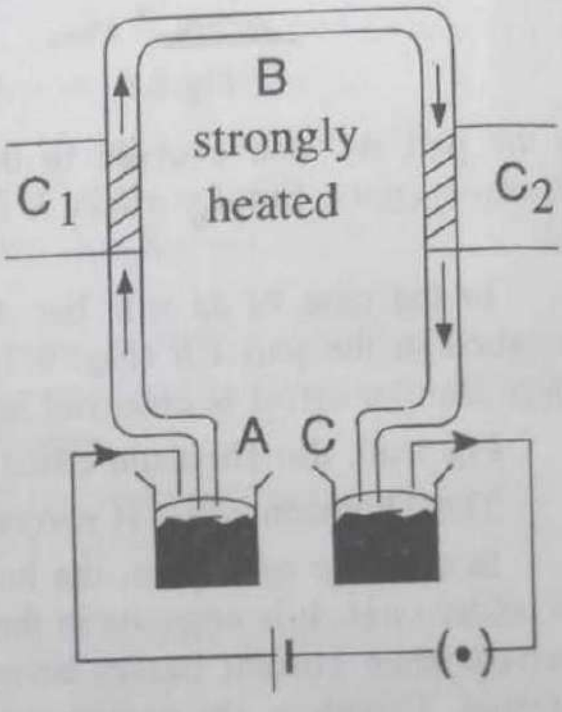

Fig. 8.8

8.6. Thermodynamics of Thermocouple

(Expressions for Peltier and Thomson coefficients.)

Consider a thermocouple consisting of two metals *A* and *B*. Let *T* and $T + dT$ be the temperatures of the cold and hot junctions respectively [Fig. 8.9]. Let π and $\pi + d\pi$ be the Peltier coefficients for the pair at the cold and hot junctions. Let σ_a and σ_b be the Thomson coefficients for the metals *A* and *B* respectively, both taken as positive. When a charge flows through the thermocouple, heat will be absorbed and evolved at the junctions due to Peltier effect and all along the metal due to Thomson effect.

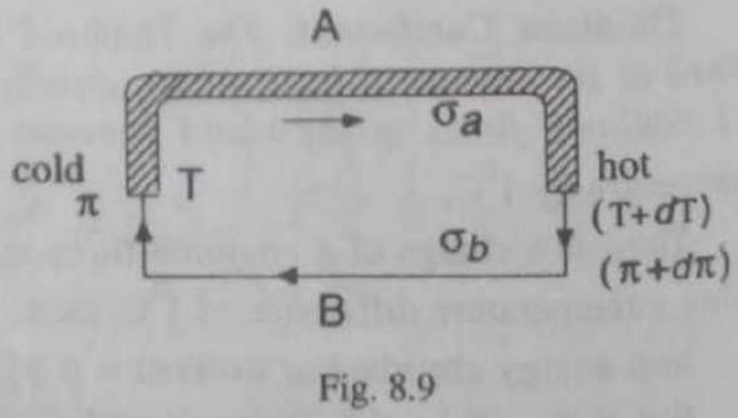

Fig. 8.9

Let 1 *coulomb of charge* flow through the thermocouple in the direction from A to B at the hot junction.

Heat energy absorbed due to Peltier effect at the hot junction = $(\pi + d\pi)$ joules

Heat energy evolved due to Peltier effect at the cold junction = π joules

Heat energy absorbed in the metal A due to Thomson effect

$$= \sigma_a \, dT \text{ joules}$$

Heat energy evolved in the metal B due to Thomson effect

$$= \sigma_b \, dT \text{ joules}$$

$\therefore$ Net heat energy absorbed in the thermocouple

$$= (\pi + d\pi - \pi) + (\sigma_a dT - \sigma_b \, dT)$$

$$= d\pi + (\sigma_a - \sigma_b) \, dT$$

This energy is used in establishing a P.D. dE in the thermocouple

$$\therefore \quad dE = d\pi + (\sigma_a - \sigma_b) \, dT \quad \text{... (1)}$$

Since the Peltier and Thomson effects are reversible, the thermocouple acts as a reversible heat engine. Here,

(*i*) the heat energy $(\pi + d\pi)$ joules is absorbed from the source at $(T + dT)$ K and $\sigma_a \, dT$ joule is absorbed in metal A at mean temperature TK.

(*ii*) Also π joule is rejected to sink at T K and $\sigma_b \, dT$ joule is given out in metal B at the mean temperature T K.

Applying Carnot's theorem, we have

$$\frac{\pi + d\pi}{T + dT} + \frac{\sigma_a \, dT}{T} = \frac{\pi}{T} + \frac{\sigma_b \, dT}{T}$$

or $$\frac{\pi + d\pi}{T + dT} - \frac{\pi}{T} = \frac{(\sigma_b - \sigma_a)dT}{T}$$

or $$\frac{\pi T + d\pi \, T - \pi T - \pi dT}{T(T + dT)} = \frac{(\sigma_b - \sigma_a)dT}{T}$$

or $$d\pi.T - \pi.dT = (\sigma_b - \sigma_a) \, dT \, (T + dT)$$

or $$d\pi.T - \pi dT = (\sigma_b - \sigma_a) \, TdT + (\sigma_b - \sigma_a) \, dT^2$$

or $$(d\pi.T - \pi dT) = (\sigma_b - \sigma_a) \, T.dT$$

[Neglecting $(\sigma_b - \sigma_a) \, dT^2$]

or $$T \, [d\pi + (\sigma_a - \sigma_b) \, dT] = \pi \, d \, T$$

But $$d\pi + (\sigma_a - \sigma_b) \, dT = dE \qquad \text{from Eq. (1)}$$

$$\therefore \quad T \, dE = \pi.dT$$

$$\frac{dE}{dT} = a + 2bT$$

dE/dT is called *thermoelectric power.*

A graph between thermoelectric power (dE/dT) and difference of temperature T is a straight line. This graph is called the *thermo-electric power line or the thermo-electric diagram.* Thomson coefficient of lead is zero. So generally thermo-electric lines are drawn with lead as one metal of the thermocouple. The thermoelectric line of a Cu-Pb couple has a positive slope while that of Fe-Pb couple has a negative slope. Fig. 8.10 shows the power lines for a number of metals.

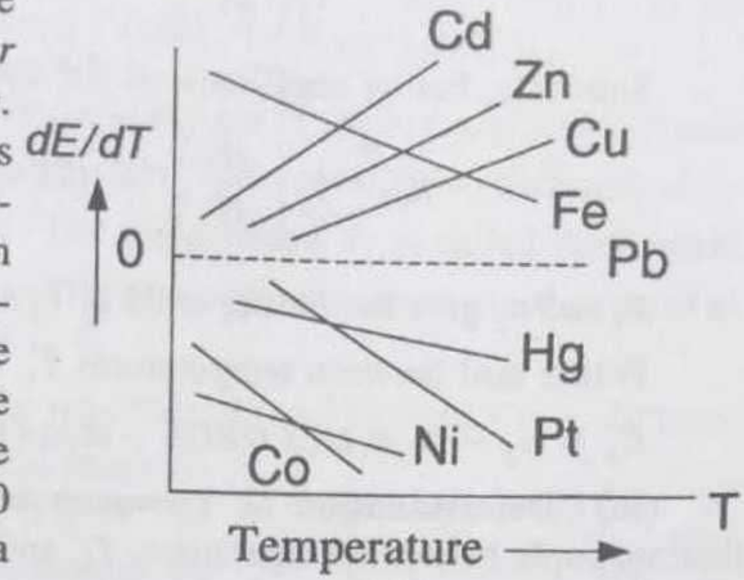

Fig. 8.10

8.8. Uses of Thermoelectric Diagrams

(*i*) **Determination of Total emf.** *MN* represents the thermo-electric power line of a metal like copper coupled with lead (Fig. 8.11). *MN* has a positive slope. Let A and B be two points corresponding to temperatures T_1 K and T_2 K respectively along the temperature-axis. Consider a small strip *abdc* of thickness dT with junctions maintained at temperatures T and $(T + dT)$.

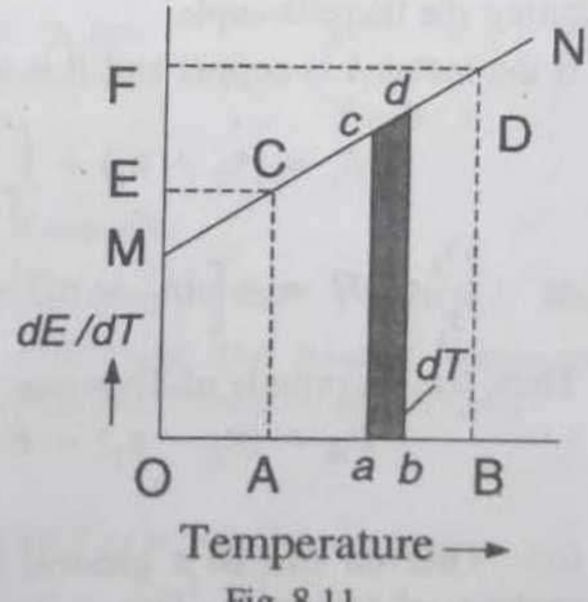

Fig. 8.11

The emf developed when the two junctions of the thermocouple differ by dT is

$$dE = dT\left(\frac{dE}{dT}\right) = \text{area } abdc$$

Total emf developed when the junctions of the couple are at temperatures T_1 and T_2 is

$$E_s = \int_{T_1}^{T_2} dT\left(\frac{dE}{dT}\right) = \text{Area } ABDC$$

(*ii*) **Determination of Peltier emf.** Let π_1 and π_2 be the Peltier coefficients for the junctions of the couple at temperatures T_1 and T_2 respectively.

The Peltier coefficient at the hot junction (T_2) is

$$\pi_2 = T_2\left(\frac{dE}{dT}\right)_{T_2} = OB \times BD = \text{area } OBDF$$

Similarly, Peltier coefficient at the cold junction (T_1) is

$$\pi_1 = T_1\left(\frac{dE}{dT}\right)_{T_1} = OA \times AC = \text{area } OACE$$

π_1 and π_2 give the Peltier emfs at T_1 and T_2 respectively.

Peltier emf between temperatures T_1 and T_2 is

$E_p = \pi_2 - \pi_1 =$ area $OBDF$ – area $OACE$ = area $ABDFECA$

(*iii*) **Determination of Thomson emf.** Total emf developed in a thermocouple between temperatures T_1 and T_2 is

$$E_S = (\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_a - \sigma_b)\, dT$$

Here σ_a and σ_b represent the Thomson coefficients of two metals constituting the thermocouple.

If the metal A is copper and B is lead, then $\sigma_b = 0$.

$$\therefore \quad E_s = (\pi_2 - \pi_1) + \int_{T_1}^{T_2} (\sigma_a\, dT)$$

or $$\int_{T_1}^{T_2} \sigma_a\, dT = -\left[(\pi_2 - \pi_1) - E\right]$$

Thus, the magnitude of Thomson emf is given by

$$E_{th} = (\pi_2 - \pi_1) - E = \text{Area } ABDFECA - \text{Area } ABDC$$
$$= \text{Area } CDFE$$

(*iv*) **Thermo emf in a general couple, neutral temperature and temperature of inversion.** In practice, a thermocouple may consist of any two metals. One of them need not be always lead. Let us consider a thermocouple consisting of any two metals, say Cu and Fe. AB and CD are the thermo-electric power lines for Cu and Fe with respect to lead (Fig. 8.12). Let T_1 and T_2 be

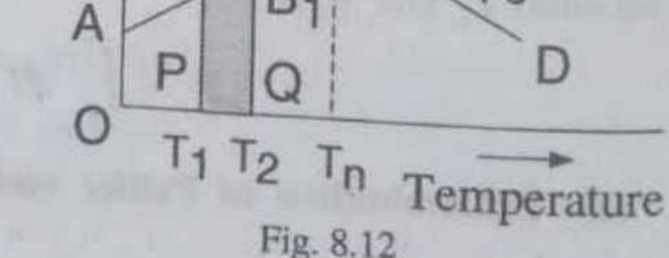

Fig. 8.12

the temperatures of the cold and hot junctions corresponding to points P and Q.

Emf of $Cu - Pb$ thermocouple = Area PQB_1A_1

Emf of $Fe - Pb$ thermocouple = Area PQD_1C_1

$\therefore$ the emf of $Cu - Fe$ thermocouple is

$$E_{cu}^{Fe} = \text{Area } PQD_1C_1 - \text{Area } PQB_1A_1 = \text{Area } A_1B_1D_1C_1$$

The emf E_{Cu}^{Fe} increases as the temperature of the hot junction is raised and becomes maximum at the temperature T_n, where the two thermoelectric power lines intersect each other. The temperature T_n is called the neutral temperature. As the thermo emf becomes maximum at the neutral temperature, at $T = T_n$, $(dE/dT) = 0$.

Suppose temperatures of the junctions, T_1 and T_2, for a $Cu - Fe$ thermocouple are such that the neutral temperature T_n lies between T_1 and T_2 (Fig. 8.13). Then the thermo emf will be represented by the difference between the areas A_1NC_1 and B_1D_1N because these areas represent opposing emf's. In the particular case when $T_n = (T_1 + T_2)/2$, these areas are equal and the resultant emf is zero. In this case, T_2 is the 'temperature of inversion' for the $Cu - Fe$ thermocouple.

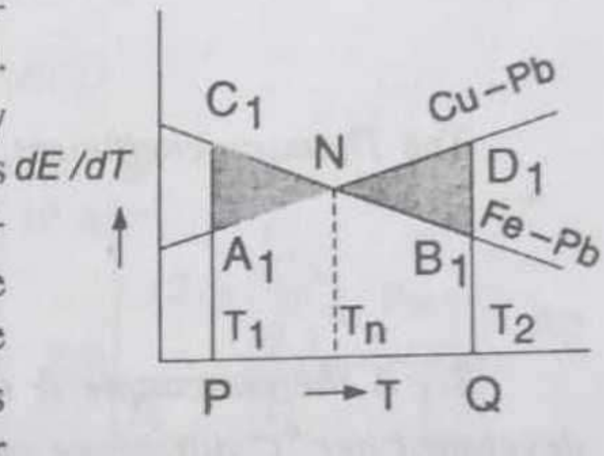

Fig. 8.13

Solved Examples