UNIT - V

THE BIOT - SAVART LAW:

Consider a conductor XY carrying a current i. Consider an element AB of length dl. O is the midpoint of AB. P is a point at a distance r from O. θ is the angle between dl and r.

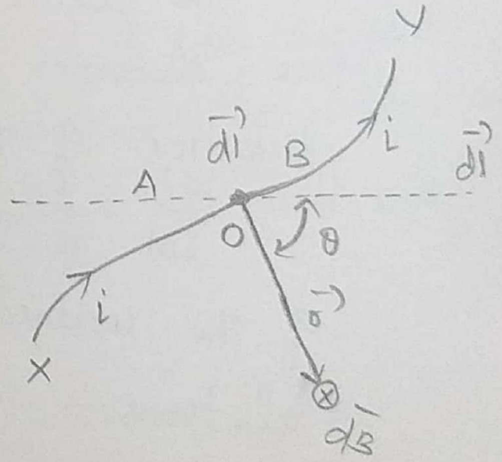
Magnetic induction dB at point P due to the current element dl is

$$dB = \left[\frac{\mu_0}{4\pi}\right] \frac{i\,(dl \times \hat{r})}{r^2}$$

Where $\hat{r}$ is a unit vector along r.

The magnitude of dB is

$$dB = \left[\frac{\mu_0}{4\pi}\right] \frac{i\,dl\,\sin\theta}{r^2}$$



The direction of dB is that of the vector $dl \times r$

The total magnetic induction B at P due to the current flowing in entire length of the conductor is

$$B = \int dB = \left[\frac{\mu_0}{4\pi}\right] \int i \frac{(dl \times \hat{r})}{r^2}$$

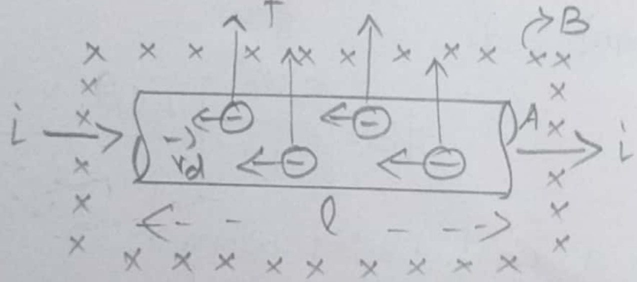
In Vacuum, B is related to H (magnetic field intensity) by the formula $B = \mu_0 H$

Where, μ_0 is a constant, called the permeability of free space.

Force on a Current - Carrying Conductor in a Magnetic Field:

Consider a straight conductor of length l and cross-sectional area A, carrying current i.

It is placed in a field of magnetic induction B at right angles to the length of the conductor and directed into the plane of the paper.



The electric current i in a conductor is due to free electrons moving in a direction opposite to the direction of the electric current.

Let n be the no. of free electrons per unit volume of the conductor and v_d the drift velocity of the electrons.

The magnitude of the force on each electron is

$$f = ev_d B. \qquad (\because v_d \perp B)$$

The no. of electrons in the length l of the conductor is

$$N = nlA.$$

Therefore, total force on all the free electrons, i.e., on the length l of the conductor is

$F = fN = (e v_d B) N$

$= (e v_d B)(n l A)$

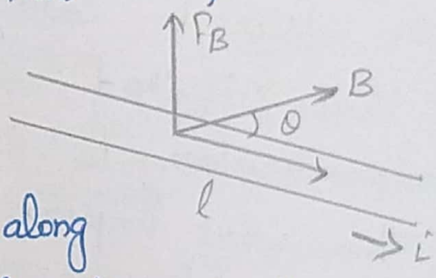
$= (n A e v_d)(B l).$

But $n A e v_d = i$, the current in the wire

$\therefore F = iBl.$

If the conductor makes an angle θ with the magnetic field induction B, then the force is

$F = iBl \sin\theta.$



Vectorially, $F = i\vec{l} \times B.$

Here, l is a vector pointing along the straight conductor in the direction of the current.

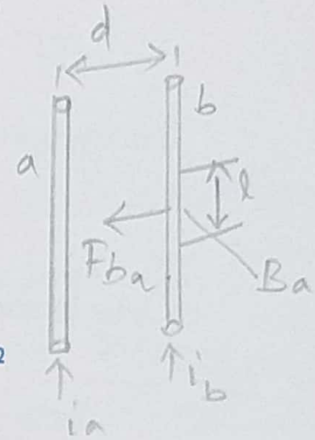
Direction of F is perpendicular to both, l and B and is given by Flemming's left hand rule.

Force between two parallel Current-Carrying conductors:

Two long parallel wires, seperated by a distance d and carrying currents i_a and i_b

magnetic induction at every point of wire b due to current i_a in wire a $\Bigg\} = B_a = \dfrac{\mu_0 i_a}{2\pi d}$ —— (1)

According to the right hand rule, the direction of B_a is downward.



Wire b, carrying the current i_b, is thus situated in a magnetic field B_a perpendicular to its length. A length l of this wire experiences a magnetic force whose magnitude is

$$F_{ba} = B_a i_b l = \frac{\mu_0 i_a i_b l}{2\pi d} \quad \text{—— (2)}$$

F_{ba} lies in the plane of the wires and points to the left.

Force experienced by a unit length of the conductor $\Bigg\} = F = \dfrac{\mu_0 i_a i_b}{2\pi d}$.

IIly, it can be shown that a unit length of wire a also experiences the same force but in the opposite direction.

Thus the two wires should attract each other.

For antiparallel currents, the two wires repel each other.

Definition of ampere

Force per unit length between two parallel and long current carrying wires $\Bigg\} = F = \dfrac{\mu_0 i_a i_b}{2\pi d}$

Put $i_a = i_b = 1A$ and $d = 1m$, $\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$.

$$F = \frac{(4\pi \times 10^{-7}) \times 1 \times 1}{2\pi \times 1}$$

$$= 2 \times 10^{-7}\ Nm^{-1}$$

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce on each of these conductors a force equal to 2×10^{-7} newtons per metre of length.

Thus the value of μ_0 may be found from the definition of the ampere.

If d, i_a, i_b are each equal to 1 and $F = 2 \times 10^{-7} N$,

$$\mu_0 = \frac{F(2\pi d)}{i_a i_b}$$

$$= \frac{(2 \times 10^{-7}) \times 2\pi \times 1}{1 \times 1} = 4\pi \times 10^{-7}\ TmA^{-1}$$

This gives us a practical method of measuring the value of μ_0.

Magnetic induction at a Point due to a Straight Conductor Carrying Current.

Consider a straight conductor XY carrying a current i in the direction Y to X.

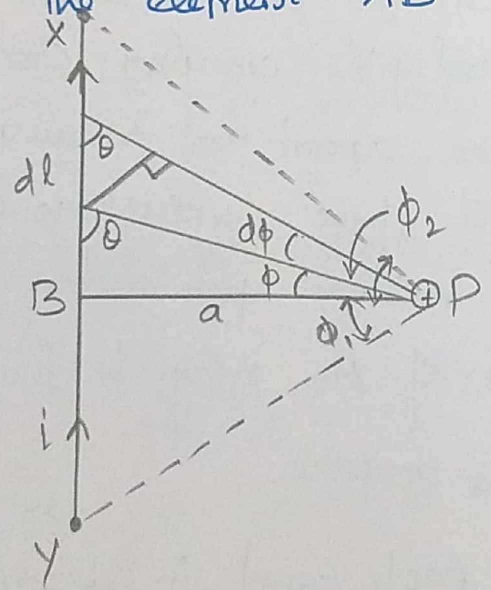
P is a point at a perpendicular distance a from the conductor.

Consider an element AB of length dl.

let $BP = r$ and $\angle OBP = \theta$

magnetic induction at P due to the element AB $\Big\} = dB$

$$= \left[\frac{\mu_0}{4\pi}\right]\frac{i\,dl\sin\theta}{r^2} \quad \text{--- (1)}$$



From B, draw BC perpendicular to PA.

let $\angle OPB = \phi$, $\angle BPA = d\phi$.

$BC = dl\sin\theta = r\,d\phi$

$$dB = \left[\frac{\mu_0}{4\pi}\right]\frac{i\,r\,d\phi}{r^2} = \left[\frac{\mu_0}{4\pi}\right]\frac{i\,d\phi}{r}$$

In $\triangle OPB$, $\cos\phi = \dfrac{a}{r}$,

$$r = \frac{a}{\cos\phi}$$

$$\therefore dB = \left[\frac{\mu_0}{4\pi}\right] \frac{i \cos\phi \, d\phi}{a} \quad \text{--- (2)}$$

The direction of dB will be perpendicular to the plane containing dl and r.

It will be directed into the page at P as shown by right hand rule.

Let ϕ_1 and ϕ_2 be the angles made by the ends of the wire at P. Then, magnetic induction at P due to the whole conductor is,

$$B = \int_{-\phi_1}^{\phi_2} \left[\frac{\mu_0}{4\pi}\right] \frac{i \cos\phi \, d\phi}{a}$$

$$= \left[\frac{\mu_0}{4\pi}\right] \frac{i}{a} \left[\sin\phi\right]_{-\phi_1}^{\phi_2}$$

$$= \left[\frac{\mu_0}{4\pi}\right] \frac{i}{a} \left[\sin\phi_2 - \sin(-\phi_1)\right]$$

$$= \left[\frac{\mu_0}{4\pi}\right] \frac{i}{a} \left[\sin\phi_2 + \sin\phi_1\right] \quad \text{--- (3)}$$

If the conductor is infinitely long, $\phi_1 = \phi_2 = 90^\circ$.

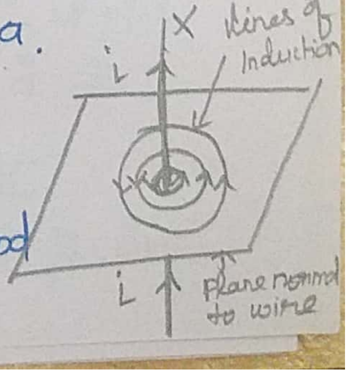
$$\therefore B = \left[\frac{\mu_0}{4\pi}\right] \frac{i}{a} [1+1]$$

$$= \frac{\mu_0 i}{2\pi a} \quad \text{--- (4)}$$

Magnitude of B depends on i and a.

$$B \propto \frac{1}{a}.$$

The lines of B form concentric circles around the wire (Fig)

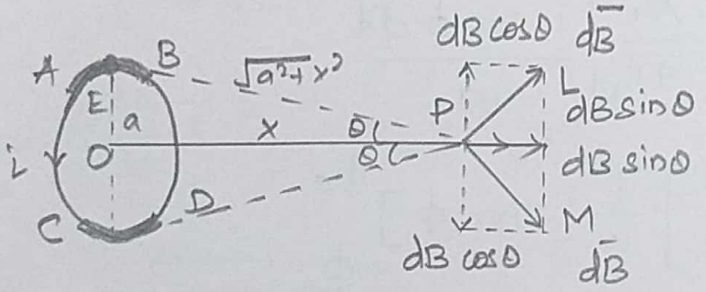


Magnetic Induction at a point on the Axis of a Circular Coil Carrying Current

Consider a circular coil of radius a, carrying a current i. P is a point on its axis at a distance x from the centre O.

Consider two opposite current elements AB and CD each of length dl.

The distance of P from any point on the circumference of the coil is $\sqrt{a^2+x^2}$.



The field at P due to AB $= dB = \left[\frac{\mu_0}{4\pi}\right]\frac{i\,dl}{(a^2+x^2)}$ ——— ①

∵ The direction of the current is at right angles to the line joining P to AB.

This is in the direction PL, perpendicular to the line joining the midpoint of AB with P.

Considering the element CD, the magnitude of dB at P due to this element is the same as that given in Equ ①. But it is directed along PM.

E is the midpoint of AB. Let $\angle EPO = \theta$.

The components dB cos θ perpendicular to the axis of the coil and due to the two opposite elements

Cancel each other. But components $dB \sin\theta$ along the axis are in the same direction.

Thus, the total magnetic induction at P due to the entire coil is

$$B = \int_{l=0}^{l=2\pi a} dB \sin\theta$$

$$= \int_{l=0}^{l=2\pi a} \left[\frac{\mu_0}{4\pi}\right] \frac{i\,dl \sin\theta}{(a^2+x^2)}$$

$$= \int_0^{2\pi a} \left[\frac{\mu_0}{4\pi}\right] \frac{i\,dl}{(a^2+x^2)} \times \frac{a}{(a^2+x^2)^{1/2}} \qquad \left[\because \sin\theta = \frac{a}{(a^2+x^2)^{1/2}}\right]$$

$$= \left[\frac{\mu_0}{4\pi}\right] \frac{i\,a}{(a^2+x^2)^{3/2}} \times 2\pi a$$

$$= \frac{\mu_0 i a^2}{2(a^2+x^2)^{3/2}} \qquad \text{— (2)}$$

If the coil has N turns, then

$$B = \frac{\mu_0 N i a^2}{2(a^2+x^2)^{3/2}} \qquad \text{— (3)}$$

At the Centre of the coil, $x = 0$

$$B = \frac{\mu_0 N i}{2a} \qquad \text{— (4)}$$

When $x >> a$,

$$B = \frac{\mu_0 N i a^2}{2x^3} \qquad \text{— (5)}$$

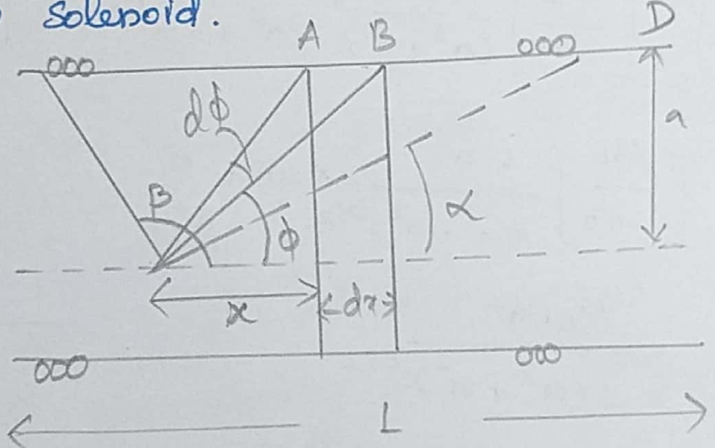
Magnetic Induction at any point on the axis of a Solenoid:

Let L represent the length of the solenoid and N the total no. of turns in its winding.

The no. of turns per unit length is then N/L.

a is the radius of the solenoid. A current i is flowing in the solenoid.

The solenoid contains air in its core. Let us find the magnetic induction B at a point P on the axis of the solenoid.



Consider an elementary length dx of the solenoid, at a distance x from P.

We can regard this element AB as a circular coil of radius a containing $N\,dx/L$ turns.

Magnetic induction at P due to the element dx is

$$dB = \frac{\mu_0 i a^2}{2} \cdot \frac{N\,dx}{L} \cdot \frac{1}{(a^2+x^2)^{3/2}} \qquad \text{—} \; (1)$$

Let us use the angle ϕ instead of x as the independent variable. Then, $x = a \cot\phi$,

$$dx = -a \operatorname{cosec}^2\phi \, d\phi$$

Substituting these values of x and dx in Eq (1),

$$dB = -\frac{\mu_0 i a^2}{2} \cdot \frac{N}{L} \cdot \frac{a \csc^2\phi \, d\phi}{[a^2 + a^2 \cot^2\phi]^{3/2}}$$

$$= -\frac{\mu_0 i N}{2L} \sin\phi \, d\phi$$

The magnetic induction at P due to the entire length of the solenoid.

$$B = -\frac{\mu_0 i N}{2L} \int_{\beta}^{\alpha} \sin\phi \, d\phi$$

$$= \frac{\mu_0 i N}{2L} [\cos\alpha - \cos\beta] \qquad \text{--- (2)}$$

The direction of B is parallel to the axis of the Solenoid.

If the core of the solenoid consists of magnetic material of permeability μ, the magnetic induction inside such a Solenoid is

$$B = \frac{\mu i N}{2L} [\cos\alpha - \cos\beta]$$

where $\mu = \mu_0 \mu_r$

$\mu = B/H$.

Special Cases:

(i) At a point well inside a very long solenoid:

$\alpha = 0, \beta = 180°$

$$B = \mu_0 i N/L \qquad \text{--- (3)}$$

(ii) At an axial point at one end of a long Solenoid: $\alpha = 0, \beta = 90°$.

$$B = \mu_0 i N/2L \qquad \text{--- (4)}$$

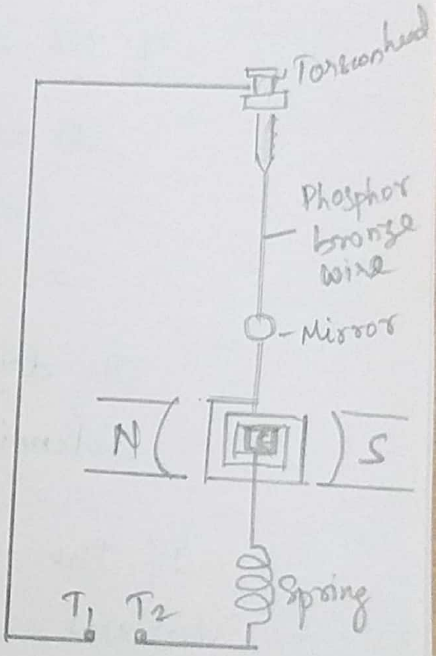
Hence the magnetic induction at either end is one-half its magnitude at points well inside the solenoid.

Moving Coil Ballistic Galvanometer:

Principle:

When a current is passed through a coil, suspended freely in a magnetic field, it experiences a force in a direction given by Fleming's left hand rule.

Construction:



It consists of a rectangular coil of thin copper wire wound on a non-metallic frame of ivory.

It is suspended by means of a phosphor bronze wire between the poles of a powerful horse-shoe magnet.

A small circular mirror is attached to the suspension wire. Lower end of the coil is connected to a hair-spring.

The upper end of the suspension wire and the lower end of the spring are connected to terminals T_1 and T_2.

A cylindrical soft iron core (C) is placed symmetrically inside the coil between the magnetic poles which are also made cylindrical in shape. This iron core concentrates the magnetic field and helps in producing radial field.

The B.G is used to measure electric charge.

The charge has to pass through the coil as quickly as possible and before the coil starts moving.

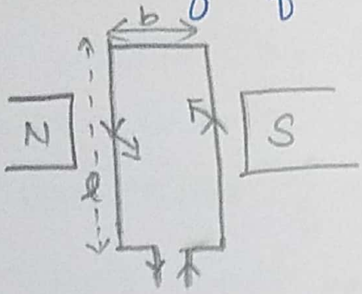
The coil thus gets an impulse and a throw is registered.

To achieve this result, a coil of high moment of inertia is used so that the period of oscillation of the coil is fairly large. The oscillations of the coil are practically undamped.

Theory:

(i) Consider a rectangular coil of N turns placed in a uniform magnetic field of magnetic induction B.

Let l be the length of the coil and b its breadth.



Area of the coil $= A = lb$.

When a current i passes through the coil,

torque on the coil $= \tau = NiBA$ — ①

If the current passes for a short interval dt, the angular impulse produced in the coil is

$$\tau\, dt = NiBA\, dt \quad —②$$

If the current passes for t seconds, the total angular impulse given to the coil is

$$\int_0^t \tau\, dt = NBA\int_0^t i\, dt = NBAq \quad —③$$

Here, $\int_0^t i\, dt = q =$ total charge passing through the galvanometer coil.

Let I be the moment of inertia of the coil about the axis of suspension and ω its angular velocity.

Change in angular momentum of the coil $= I\omega$ — ④

$$\therefore I\omega = NBAq \quad —⑤$$

(ii) The kinetic energy of the moving system $\frac{1}{2}I\omega^2$ is used in twisting the suspension wire through an angle θ.

Let C be the restoring torque per unit twist of the suspension wire. Then,

work done in twisting the suspension wire by an angle $\theta = \frac{1}{2}C\theta^2$

$$\therefore \frac{1}{2}I\omega^2 = \frac{1}{2}C\theta^2.$$

or $\quad I\omega^2 = c\theta^2 \qquad — (6)$

(iii) The period of oscillation of the coil is

$$T = 2\pi\sqrt{\left(\frac{I}{c}\right)}$$

$$T^2 = \frac{4\pi^2 I}{c}$$

$$I = \frac{T^2 c}{4\pi^2} \qquad — (7)$$

Multiplying Equ (6) and (7),

$$I^2\omega^2 = \frac{c^2 T^2 \theta^2}{4\pi^2}$$

$$I\omega = \frac{cT\theta}{2\pi} \qquad — (8)$$

Equ (5) and (8), $NBAq = \dfrac{cT\theta}{2\pi}$

or

$$q = \left[\frac{T}{2\pi}\right]\left[\frac{c}{NBA}\right]\theta \qquad — (9)$$

This gives the relation between the charge flowing and the ballistic throw θ of the galvanometer. $q \propto \theta$.

$\left[\frac{T}{2\pi}\right]\left[\frac{c}{NBA}\right]$ is called the ballistic reduction factor (k).

$$\therefore \; q = k\theta \qquad — (10).$$

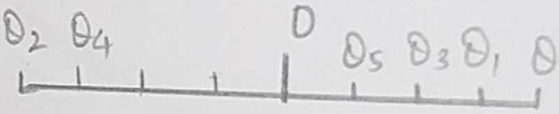
Correction for Damping in Ballistic Galvanometer:

We have assumed that the whole of the K.E imparted to the coil is used in twisting the suspension of the coil. In actual practice, the motion of the coil is damped by air resistance and the induced current produced in the coil.

The first throw of the galvanometer is, therefore, smaller than it would have been in the absense of damping. The correct value of first throw is however obtained by applying damping correction.

Let $\theta_1, \theta_2, \theta_3 \ldots$ be the successive maximums deflections from zero position to the right and left.

Then it is found that



$$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = \ldots = d$$

The constant d is called the decrement per half vibration.

Let $d = e^{\lambda}$ so that $\lambda = \log_e d$.

Here λ is called the logarithmic decrement.

For a complete vibration,

$$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda}.$$

Let θ be the true first throw in the absence of damping.

$\theta > \theta_1$, the first throw θ_1 is observed after the coil completes a quarter of vibration.

In This case, the value of the decrement would be $e^{\lambda/2}$.

$$\therefore \frac{\theta}{\theta_1} = e^{\lambda/2} \approx \left[1 + \frac{\lambda}{2}\right].$$

$$\theta = \theta_1 \left[1 + \frac{\lambda}{2}\right] \qquad -(2)$$

We can calculate λ by observing the first throw θ_1 and the eleventh throw θ_{11}.

$$\frac{\theta_1}{\theta_{11}} = \frac{\theta_1}{\theta_2} \cdot \frac{\theta_2}{\theta_3} \cdot \frac{\theta_3}{\theta_4} \cdot \frac{\theta_4}{\theta_5} \cdot \frac{\theta_5}{\theta_6} \cdot \frac{\theta_6}{\theta_7} \cdot \frac{\theta_7}{\theta_8} \cdot \frac{\theta_8}{\theta_9} \cdot \frac{\theta_9}{\theta_{10}} \cdot \frac{\theta_{10}}{\theta_{11}} = e^{10\lambda}$$

$$\lambda = \frac{1}{10} \log_e \frac{\theta_1}{\theta_{11}}$$

$$= \frac{2.3026}{10} \log_{10} \frac{\theta_1}{\theta_{11}}. \qquad -(3)$$

$$\therefore q = \left[\frac{T}{2\pi}\right]\left[\frac{C}{NBA}\right]\theta_1 \left[1 + \frac{\lambda}{2}\right] \qquad -(4).$$

Galvanometer are classified as (i) dead-beat or aperiodic
(ii) ballistic galvanometer.

A moving coil galvanometer in which the coil is wound on a metallic conducting frame is known as a dead-beat galvanometer. It is called "dead-beat" because it gives a steady deflection without producing any oscillation, when a steady current is passed through the coil.