Viscosity

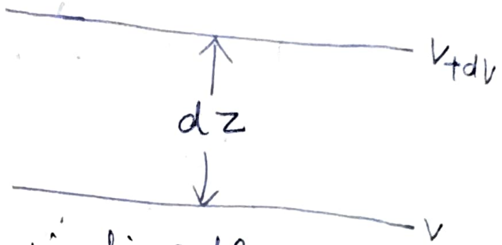
When a liquid flows slowly and steadily through a pipe, the layer of liquid in contact with the walls of the pipe remains practically stationary. The layer of liquid along its axis which is farthest from the stationary layer moves with the greatest velocity. The intermediate layers moves with different velocities. The velocity of the layers increases from zero at the walls of the pipe to a maximum along its axis. There is a varation of velocity with distance or a velocity gradient is set up.

If we consider any particular layer of a liquid we find that the layer immediately above it is moving faster than the layer immediately below it. Hence the upper layer tends to increase the velocity of the lower layer. whereas the ~~velocity~~ lower layer tends to decrease the velocity of the upper layer. The two layers together tend to destroy their relative motion as if there is some Backward Dragging Force acting tangentially on the layers. This tangential backward dragging force, comming into play in between two adjacent layers of a liquid and tending to oppose the relative motion between them, is called the

Viscous force: This property by virtue of which a liquid opposes relative motion between its different layers is called viscosity. Hence if a relative velocity (movement) between the layers of a liquid is to be maintained, an external force must be applied to it to overcome the viscous force.

Newton's law of Viscous Flow:

Consider two layers of liquid separated by a distance dz. Let v and ~~$v+dv$~~ $v+dv$ be the velocity of two layers. So the velocity gradient is dv/dz.



Let A be the area of the layer. According to Newton, the viscous force is directly proportional to the surface area A and velocity gradient dv/dz

i.e, $F \propto A \frac{dv}{dz}$ or $F = \eta A \frac{dv}{dz}$

where η is a constant for the liquid called coefficient of viscosity. This is known as Newton's law of viscous flow in liquids.

Definition of coefficient of viscosity: In the above relation, if $A=1$ and $\frac{dv}{dz}=1$, we have $F=\eta$. The coefficient of viscosity is defined as the tangential force per unit area required to maintain a unit velocity gradient.

Unit of coefficient of viscosity: The SI unit is newton second per square metre (Nsm^{-2}).

Dimensions of coefficient of viscosity

The coefficient of viscosity is given by the relation, $\eta = \frac{F}{A(dV/dz)}$

Dimension of F : $F = ma$, $V = \frac{d}{t}$, $a = \frac{dv}{dt}$ $[v]$

V - velocity

$\therefore$ Dimension of acceleration (a) $= \frac{1}{T} \times \frac{L}{T}$

$\therefore F = MLT^{-2}$

Dimension of Area : Area $= L^2$

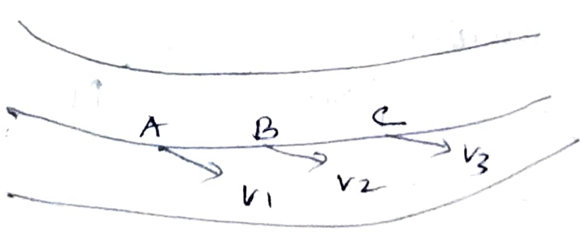
Dimension of velocity gradient $\frac{dv}{dz} = \frac{LT^{-1}}{L} = T^{-1}$

Dimension of $\eta = \frac{MLT^{-2}}{L^2T^{-1}} = ML^{-1}T^{-1}$

=

Stream line Flow and Turbulent flow.

Consider a liquid flowing in a pipe. Let the velocity of flow be V_1 at A, V_2 at B and V_3 at C. If as time passes the velocities at A, B and C are constant in magnitude and direction, the flow is said to be steady. In a steady flow, each particle follows exactly the same path and has exactly the same velocity as its predecessor. Now the liquid is said to have an ordinary or stream line flow.



The line ABC is called the stream line. The tangent to the streamline at any point gives the velocity of the liquid at that point.

The flow is steady or stream-lined only as long as the velocity of the liquid does not exceed a limiting value called the critical velocity. When the external pressure causing the flow of liquid is excessive, the motion of the liquid takes place with a velocity greater than the critical velocity and the motion becomes unsteady or turbulent. This causes eddies and whirlpools in the motion of the liquid.

Definition of critical velocity: Critical velocity of a liquid is the velocity below which the motion of the liquid is orderly and above which the motion of the liquid becomes turbulent.

Deduction of Expression for the critical velocity by the method of Dimensions: The critical velocity of a liquid may depend on (i) the coefficient of viscosity of the liquid (ii) the density of the liquid (ρ) and (iii) the radius r of the tube through which the liquid is flowing. We may write

$$V_c = K \eta^a \rho^b r^c$$

where K is a dimensionless number called Reynold's number. Writing the dimensions of these quantities

$$LT^{-1} = [ML^{-1}T^{-1}]^a [ML^{-3}]^b [L]^c$$

comparing the coefficients of a, b, c

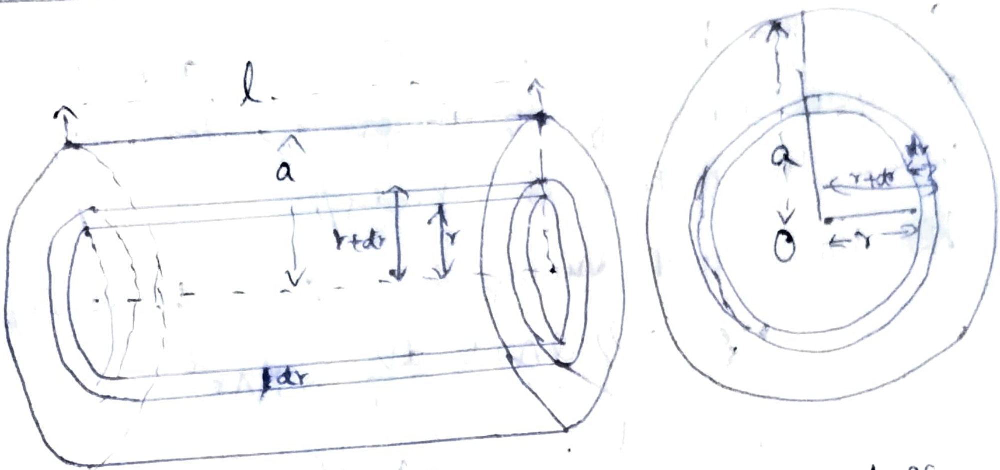
$a + b = 0$, $-a - 3b + c = 1$ and $-a = -1$

$\therefore$ $a = 1$, $b = -1$ and $c = -1$

$$V_c = \frac{K\eta}{\rho r}$$ (For narrow tubes K = 1000)

Then the critical velocity of the liquid is directly proportional to its viscosity (η), inversely proportional to its density and the radius of the tube

Poiseuille's formula for the flow of liquid through a capillary tube :



Suppose a constant pressure difference P is maintained between the two ends of the cappilary tube of length l and radius a as shown in fig. Consider the steady flow of liquid of coefficient of viscosity η through the tube. The velocity of the liquid is maximum along the axis and is zero at the walls of the tube. Assume that there is no radial flow. Consider a cylindrical shell of the radius r and outer radius r+dr. Let the velocity of the liquid on the inner surface of the shell be v and that on the outer surface of the shell be v-dv. So that $-dv/dr$ is the velocity gradient.

The surface area of the cell $A = 2\pi r l$

According to Newton's law of viscous flow, the backward dragging tangential force exerted by the outer layer on the inner layer opposite to the direction of motion $F_1 = -\eta A \frac{dv}{dr}$

$$F_1 = -\eta 2\pi r l \frac{dv}{dr}$$

The driving force on the liquid shell, accelerating it forward, $F_2 = P\pi r^2$ where P = pressure difference across the two ends of the tube and πr^2 = Area of cross section of the inner cylinder.

When the motion is steady,

the backward dragging force (F_1) = the driving force (F_2)

i.e. $$-\eta 2\pi r l \frac{dv}{dr} = P\pi r^2$$

or $$dv = \frac{-P}{2\eta l} r\, dr$$

Integrating on both sides $v = \frac{-P}{2\eta l}\frac{r^2}{2} + C$

where C is a constant of integration

when $r = a$, $v = 0$

$$-\frac{P}{2\eta l}\frac{a^2}{2} + C = 0$$

$$C = \frac{Pa^2}{4\eta l}$$

$$v = \frac{-P}{2\eta l}\frac{r^2}{2} + \frac{Pa^2}{4\eta l}$$

$$\therefore v = \frac{P}{4\eta l}(a^2 - r^2)$$

This gives the average velocity of the liquid flowing through the cylindrical shell.

Hence the volume of the liquid that flows out per second through the shell = dV

$dV = \left(\begin{array}{l}\text{Area of cross section of the shell} \\ \text{of radius } r \text{ and thickness } dr\end{array}\right) \times$ velocity of flow

$$= 2\pi r\,dr \times \frac{P}{4\eta l}(a^2 - r^2)$$

$$dV = \frac{\pi P}{2\eta l}(a^2 r - r^3)\,dr$$

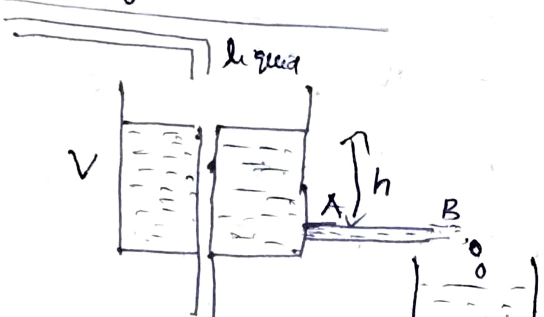
The volume of liquid that flows out per second ~~through the~~ is obtained by integrating the expression for dV between the limits $r = 0$, to $r = a$

$$\therefore V = \frac{\pi P}{2\eta l}\left[\frac{a^2 r^2}{2} - \frac{r^4}{4}\right]_0^a$$

$$= \frac{\pi P}{2\eta l} \times \frac{a^4}{4}$$

$$\boxed{V = \frac{\pi P a^4}{8\eta l}}$$

Poiseuille's method for determining coefficient of viscosity of a liquid



The liquid is taken in the constant level tank up to a height h. A capillary tube AB is fixed at the bottom of the ~~flask~~ tank. A weighed beaker is placed below the free end B of the capillary tube and the weight w of the liquid collected in it in time 't' seconds is found out.

Volume of liquid flowing per second $V = \frac{w}{\rho t}$ where ρ is the density of the liquid. The length l of the capillary tube is measured by metre rod. The radius of capillary tube is determined very accurately using the travelling microscope. Then from the relation $\eta = \frac{\pi P a^4}{8Vl}$ the value of η for the liquid can be calculated where $P = h\rho g$.

Highly viscous liquids

Stokes Formula: The viscous force experienced by a falling sphere must depend on

(i) the terminal velocity (v) of the ball

(ii) the radius (r) of the ball and

(iii) the coefficient of viscosity (η) of the liquid

We can write $F = k v^a r^b \eta^c$ where k is a dimensionless constant. The dimensions of these quantities are

$F = MLT^{-2}$, $V = LT^{-1}$, $r = L$ and $\eta = ML^{-1}T^{-1}$

$$\therefore \quad MLT^{-2} = (LT^{-1})^a L^b (ML^{-1}T^{-1})^c$$

$$MLT^{-2} = M^c L^{a+b-c} T^{-a-c}$$

Equating the powers of M, L and T on ~~at~~ either side

$c = 1$, $a+b-c = 1$ and $-a-c = -2$. Solving

$a = 1$, $b = 1$ and $c = 1$ $\therefore$ $F = kvr\eta$.

Stokes experimentally found the value of k to be 6π.

Hence $F = 6\pi\eta vr$. This is Stokes law.

Expression for terminal velocity

Let us consider an infinite column of a liquid which is highly viscous like castor oil contained in a tall jar. If a steel ball is dropped into the liquid, it begins to move down with the acceleration under gravitational pull. But its motion in the liquid is opposed by viscous forces in the liquid. These viscous forces increase as the velocity of the ball increases. Finally a velocity will be attained when the apparent weight of the ball becomes equal to the retarding viscous forces acting on it. At this stage the resultant force on the ball is zero. Therefore the ball continues to move down with the same velocity thereafter. This uniform velocity is called the terminal velocity.

Let ρ be the density of the ball and ρ' the density of the liquid.

Then, the weight of the ball $= \frac{4}{3}\pi r^3 \rho g$

The weight of the displaced liquid or the upthrust on the ball $= 4/3\,\pi r^3 \rho' g$

The apparent weight of the ball $= 4/3\,\pi r^3 \rho g - 4/3\,\pi r^3 \rho' g$

$= \frac{4}{3}\pi r^3 (\rho - \rho')g$

when the ball attains its terminal velocity v,

the apparent weight of the ball = Viscous force F

$$6\pi\eta v r = 4/3\,\pi r^3(\rho - \rho')g$$

$$\boxed{v = \frac{2}{9}\frac{r^2}{\eta}(\rho - \rho')g}$$