1 ~~M.S.R = 0.05~~ cm

Free vibrations of a body :

When a body free to oscillate is displaced from its equilibrium position and no external driving or resisting force is acting on it, it continues to oscillate with a constant amplitude and its own natural frequency. Such vibrations of a body are called free vibrations.

Example: A simple pendulum oscillating in vacuum. The simple pendulum vibrates with a time period T given by $T = 2\pi\sqrt{l/g}$. T depends only on l and g. If l and g remains constant, the pendulum will continue to oscillate with the same period and amplitude for any length of time. In all similar cases, the vibrations will be undamped free vibrations.

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Differential equation and frequency of oscillation for free undamped vibrations

For a simple harmonically vibrating particle, the kinetic energy for displacement y, is given by

$F = -\Delta V$

$F = -\frac{dV}{dx}$

$$KE = \frac{1}{2} m \left(\frac{dy}{dt}\right)^2$$

$dV = -F\,dx$

Potential energy of the particle is given by

$dV = -(-kx)\,dx$

$$PE = \frac{1}{2} k y^2$$

Where k is the restoring force per unit displacement.

$\int dV = \frac{kx^2}{2}$

The total energy at any instant

$V = \frac{1}{2}kx^2$

$$= \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} k y^2$$

For an undamped harmonic oscillator this total energy remains constant

$$\therefore \frac{1}{2} m \left(\frac{dy}{dt}\right)^2 + \frac{1}{2} k y^2 = \text{constant} \quad \text{——— (1)}$$

differentiating eqn (1) w.r.to time

$$m \frac{d^2y}{dt^2} + ky = 0 \quad \text{——— (2)}$$

$$\boxed{\frac{d^2y}{dt^2} + \frac{k}{m} y = 0} \quad \text{——— (3)}$$

Equation 3 is similar to the equation

$$\boxed{\frac{d^2y}{dt^2} + \omega^2 y = 0} \quad \text{——— (4)}$$

here $\omega^2 = k/m$

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The solution for equation (4) is

$$y = a \sin(\omega t - \alpha)$$

$$= a \sin\left[\sqrt{k/m}\, t - \alpha\right]$$

~~y = a sin [~~

The frequency of oscillation is

$$\boxed{n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}}$$

Thus in the case of undamped free vibrations, the differential equation is

$$\boxed{\frac{d^2y}{dt^2} + \left(\frac{k}{m}\right)y = 0}$$

Newton's formula for velocity of sound

The velocity of sound in a medium - solid, liquid or gas, depends on the elasticity and density of the medium

$$U = \sqrt{\frac{E}{\rho}}$$

U - velocity of sound in the medium
E - Elasticity of the medium
ρ - density of the medium

Newton assume that the temperature remains constant when sound waves travels through air. This process is isothermal and Boyle's law can be applied.

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Beats

When two sounding bodies of nearly the same frequency and amplitude are sounded together, they combine to form a single note whose intensity alternatively rises and falls at regular intervals. This phenomenon of waxing and waning of sound is called beats. The number of beats heard per second is equal to the difference in frequency between the two sounding bodies. The human ear cannot perceive more that 16 beats per second.

Expression for ~~Analytical treatment of~~ beats.

Consider two sound wave of the same amplitude 'a' having slightly different frequencies n_1 and n_2. The difference in frequency of the two waves is ν i.e $n_2 = n_1 + \nu$.

The resultant wave $y = y_1 + y_2$

$$y_1 = a \sin \frac{2\pi}{\lambda_1}(ct - x)$$

$$= a \sin 2\pi \left(n_1 t - \frac{x}{\lambda_1} \right) \qquad c/\lambda_1 = n_1$$

$$y_2 = a \sin \frac{2\pi}{\lambda_2}(ct - x)$$

$$= a \sin 2\pi \left(n_2 t - \frac{x}{\lambda_2} \right) \qquad c/\lambda_2 = n_2$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$y = a \sin \frac{2\pi}{\lambda_1}(ct - x) + a \sin \frac{2\pi}{\lambda_2}(ct - x)$$

$$= 2a \sin \tfrac{1}{2}\left\{ \left[\frac{2\pi}{\lambda_1}(ct-x) + \frac{2\pi}{\lambda_2}(ct-x) \right] \right\}$$
$$\cos \tfrac{1}{2}\left\{ \left[\frac{2\pi}{\lambda_1}(ct-x) - \frac{2\pi}{\lambda_2}(ct-x) \right] \right\}$$

$$= 2a \sin \frac{2\pi}{2}\left\{ \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right)(ct - x) \cos \frac{2\pi}{2}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)(ct - x) \right\}$$

$$\frac{1}{\lambda_1} + \frac{1}{\lambda_2} = \frac{1}{\lambda} + \frac{1}{\lambda + d\lambda}$$

$$= \frac{\lambda + d\lambda + \lambda}{\lambda(\lambda + d\lambda)} = \frac{2\lambda + d\lambda}{\lambda^2 + \lambda d\lambda}$$

$$= \frac{2\lambda}{\lambda^2} = \frac{2}{\lambda}$$

$$\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{\lambda_1} - \frac{1}{\lambda_1 + d\lambda}$$

$$= \frac{\lambda_1 + d\lambda - \lambda_1}{\lambda_1(\lambda_1 + d\lambda)} = \frac{d\lambda}{\lambda_1^2 + \lambda_1 d\lambda}$$

$$\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{\lambda} - \frac{1}{\lambda + d\lambda}$$

$$= \frac{\lambda + d\lambda - \lambda}{\lambda(\lambda + d\lambda)} = \frac{d\lambda}{\lambda^2 + \lambda d\lambda}$$

$$= \frac{d\lambda}{\lambda^2}$$

$$= 2a \sin\left(\frac{2\pi}{2} \times \frac{2}{\lambda}\right)(ct-x) \cos\left(\frac{2\pi}{2} \times \frac{d\lambda}{\lambda^2}\right)(ct-x)$$

$$= 2a \cos \frac{2\pi d\lambda}{2\lambda^2} (ct-x) \sin \frac{2\pi}{\lambda} (ct-x)$$

$$y = 2a \cos \frac{2\pi}{\lambda'} (ct-x) \sin \frac{2\pi}{\lambda} (ct-x)$$

where $\lambda' = \frac{2\lambda^2}{d\lambda}$. Then the resultant wave has amplitude which has the wave character λ'

Then the ~~wave~~ resultant wave of wavelength λ advances with periodic vibration of amplitude $\lambda' = \frac{2\lambda^2}{d\lambda}$ advances with the same velocity c as the original wave.

The distance between two consecutive maxima or minima is $\lambda'/2 = \frac{2\lambda^2}{2d\lambda} = \frac{\lambda^2}{d\lambda} = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}$

The time interval between two maxima or minima is given by $\frac{\text{distance}}{\text{velocity}} = \frac{\lambda_1\lambda_2}{c(\lambda_2-\lambda_1)}$

Hence the frequency of the occurrence of beat is

$$\frac{c(\lambda_2-\lambda_1)}{\lambda_1\lambda_2} = c\left(\frac{1}{\lambda_1} - \cancel{\frac{1}{\lambda_2}}\right) c\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= n_1 - n_2$$

Hence the number of beats heard per second is equal to the difference between the frequencies.

Then beats produce the periodic fluctuation of loudness of sound.

Uses of beats

1. To determine the frequency of a tuning fork

Take a standard tuning fork A of frequency N. The frequency of the tuning fork B is unknown. The two tuning forks are sounded together. Find the number of beats produced per second. Let it be n. Then the frequency of B is $N \pm n$. The fork B is now slightly loaded with wax thus reducing its frequency. The two forks are now sounded together. If the number of beats per second increases, the frequency of B is $N-n$. If the number of beats per second decreases, frequency of B is $N+n$.

2. Beats are made use of to tune two notes to be in unison. For example, the note produced by a sonometer wire can be tuned to be in unison with that of the tuning fork by observing beats. If the frequencies of the sounds emitted by them are not equal, then beats will be heard. The length of the sonometer wire is adjusted until the beats disappear. The frequencies of the notes emitted by the sonometer wire and the fork are exactly equal.