'AEC'

TENSORS

Introduction.

Scalars are specified by magnitude only. Vectors have magnitude as well as direction. Tensors are associated with magnitude and two or more directions.

- Scalar : Volume, mass, length, speed, temperature, potential, electric charge etc

Vector : velocity, acceleration, momentum, force, electric or magnetic field intensities etc what is the relation between tensors and physics.

In many physical problems vector notation is not sufficiently general to express the relations between quantities involved. For example let us consider the flow of electric current between current density vector J and the electric field vector E is given by vector equation.

$$J = \sigma E \quad \ldots (1)$$

Where σ is electrical conductivity of the medium. For such a medium the conductivity σ is scalar and for the x - component we write

$$J_1 = \sigma E_1 \quad \ldots (2)$$

However, if the medium is anisotropic as in many crystals, the current density in x directions may depend on the electric field in the y and z - directions as well as on the field in x - direction. Assuming a linear relationship we may replace equation (2) and write

$$J_1 = \sigma_{11} E_1 + \sigma_{12} E_2 + \sigma_{13} E_3 \quad \ldots (3)$$

and in General $J_1 \sum_K \sigma_{iK} E_K \quad \ldots (4)$

Anisotropic : Having different physical properties in different directions.

Thus, for ordinary three-dimensional space, the Scalar quantity σ must be replaced by a more general quantity σ_{ik} (a set of nine elements) to effect the required change in direction and magnitude

$$\sigma_{ik} \rightarrow \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

This general quantity σ_{ik} [array of n elements) is called a tensor. The description of electric, optical electrical and magnetic properties of anisotropic Solid may well involve tensors.

∗. The fundamental laws of physics must posses the Same form in all Coordinate systems. The study of this accepted fact, known as invarient formulation of physical laws is utilized ~~embodied~~ in tensor analysis.

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n dimensional Space.

"In three dimensional space a point is determined by a set of three numbers called the Coordinates of the point."

For example:

(x, y, z) are the coordinates of a point in rectangular Cartesian coordinates system. If a point is represented by a set of n real variables $(x^1, x^2, x^3 \ldots x^n)$ (Here the suffixes $(1, 2, 3 \ldots n)$ denotes variables and not the powers of the variables involved) then all the points corresponding to all values of coordinates (variables) are said to form an n dimensional space, denoted by V_n.

Superscripts and subscript.

The suffix ν and μ in A^{ν}_{μ} are called superscript and subscript respectively. The upper position of suffix always denotes superscript: while the lower position always denotes subscript.

Coordinates: To describe the configuration of a system, we select the smallest possible number of variables

$(A^{\nu}$ = index

$A^{\mu\nu\sigma}$ = indices.

x^i It is important to note that in writing x^i

i is merely a superscript on x and does not denote

the i the power of x.)

It is always to be kept in mind that the suffixes

μ in the coordinates x^{μ} do not have the character of power indices. Usually powers will be indicated by the use of brackets. Thus $(x^{\mu})^2$ means square of x^{μ}.

The reason for using superscripts will be indicated in due course.

Coordinate transformations:

Tensor analysis is immediately connected with the subject of coordinate transformations.

Consider two sets of variables $(x^1, x^2, x^3 \ldots x^n)$ and $\bar{x}^1, \bar{x}^2, \bar{x}^3 \ldots \bar{x}^n)$ which determine the coordinates of a point in an n-dimensional space in two different frames of reference. Let the two sets of variables be related to each other by the transformations.

$$\bar{x}^1 = \phi^1 (x^1, x^2, x^3 \dots x^n)$$

$$\bar{x}^2 = \phi^2 (x^1, x^2, x^3 \dots x^n)$$

$$\dots\dots\dots\dots\dots$$

$$\dots\dots\dots\dots\dots$$

$$\bar{x}^n = \phi^n (x^1, x^2, x^3 \dots x^n)$$

or briefly $\bar{x}^\mu = \phi^\mu (x^1, x^2, x^3 \dots x^n) \; (\mu = 1, 2 \dots n) \dots (1)$

where functions ϕ^μ are single valued, continious differentia-ble functions of coordinates. It is essential that the n-funtions ϕ^μ be independent.

Equ (1) can be solved for coordinates x^μ as functions of $\bar{x}^\mu$ to yield

$$\bar{x}^\mu = \psi^\mu (\bar{x}^1, \bar{x}^2, \bar{x}^3 \dots \bar{x}^n) \quad \dots (2)$$

Equ (1) and (2) are said to define coordinate transformations.

From equ (1) the differentials $d\bar{x}^\mu$ are transformed as

$$d\bar{x}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^1} dx^1 + \frac{\partial \bar{x}^\mu}{\partial x^2} dx^2 \dots + \frac{\partial \bar{x}^\mu}{\partial x^n} dx^n$$

$$\boxed{d\bar{x}^\mu = \sum_{\alpha=1}^{n} \frac{\partial \bar{x}^\mu}{\partial x^\alpha} dx^\alpha} \quad (\mu = 1, 2, 3 \dots n) \dots (3)$$

This is transformation law

Indicial and Summation Convention.

Let us now introduce the following two Conventions.

(1) Indicial Convention: Any Index, used either as subscript or superscript will take all values from 1 to n unless the contrary is specified. Thus equ (1) can be briefly written as

$$\bar{x}^{\mu} = \phi^{\mu}(x^{\alpha}) \quad \cdots (4)$$

The convention reminds us that there are n equations with $\mu = 1, 2, \ldots n$ and ϕ^{μ} are the functions of n Coordinates x^{α}, with $\alpha = 1, 2 \ldots n$

(2) Einstein's Summation Convention. If any index is repeated in a term, then a summation with respect to that index over the range $1, 2 \ldots n$ is implied. This Convention is called Einstein's Summation Convension.

According to this Convention instead of expression $\sum_{\mu=1}^{n} a_{\mu} x^{\mu}$, we merely write $a_{\mu} x^{\mu}$

Using these two Convention equation (3) may be written as $d\bar{x}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} dx^{\alpha}$ . . . . (5a)

Thus the summation convention means the drop of Sigma sign for the index appearing twice in a given term. In other words, the Summation Convention implied the sum of the term for the index appearing twice in that term over defined range.

Dummy and real indices:

"Any index which is repeated in a given term, So that the Summation Convention applies, is called a dummy index", and it may be replaced freely by any other index not already used in equation (5a) So that equation (5a) is equally written as.

$$d\bar{x}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\beta}} dx^{\beta} = \frac{\partial \bar{x}^{\mu}}{\partial x^{k}} dx^{k} \qquad a^{\alpha}_{\mu} x^{\mu}$$

Also two or more dummy indices Can be interchanged. In order to avoid Confusion the Same index must not be used more than twice in any single term. For example $\left(\sum_{\mu} a_{\mu} x^{\mu}\right)^{2}$ will not be written as

$$a_\mu x^\mu . a_\mu x^\mu \text{ but rather } a_\mu a_\nu x^\mu x^\nu$$

Any index which is not repeated in a given term is called a real index. For example. μ is a real index in $a_\alpha^\mu x^\alpha$. A real index can not be replaced by another real index e.g

$$a_\alpha \mu \bar{x}^\alpha \neq a_\alpha^\nu x^\alpha$$

Kronecker delta symbol.

Kronecker delta symbol δ_ν^μ is defined as

$$\delta_\nu^\mu = \begin{cases} 1 & \text{if } \mu = \nu \\ 0 & \text{if } \mu \neq \nu \end{cases} \quad \text{- - - - (6)}$$

Some properties of Kronecker delta.

(i) If $x^1, x^2, x^3 \ldots x^n$ are independent variables, then $\frac{\partial x^\mu}{\partial x^\nu} = \delta_\nu^\mu$ - - - - - - - (7)

(ii) An obvious property of Kronecker delta symbol is

$$\delta_\nu^\mu A^\mu = A^\nu \quad \text{- - - - - (8)}$$

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(iii) If we are dealing with n- dimensions, then

$$\delta^{\mu}_{\mu} = \delta^{\nu}_{\nu} = n \quad \ldots \quad (9)$$

By summation Convention

$$\delta^{\mu}_{\mu} = \delta^{1}_{1} + \delta^{2}_{2} + \delta^{3}_{3} + \ldots \delta^{n}_{n}$$

$$= 1 + 1 + 1 + \ldots = n$$

(iv) $\delta^{\mu}_{\nu}\,\delta^{\nu}_{\sigma} = \delta^{\mu}_{\sigma}$

By Summation Convention

$$\delta^{\mu}_{\nu}\,\delta^{\nu}_{\sigma} = \delta^{\mu}_{1}\delta^{1}_{\sigma} + \delta^{\mu}_{2}\delta^{2}_{\sigma} + \delta^{\mu}_{3}\delta^{3}_{\sigma} + \ldots + \delta^{\mu}_{\mu}\,\delta^{\mu}_{\sigma} + \delta^{\mu}_{n}\,\delta^{n}_{\sigma}$$

$$= 0 + 0 + 0 + \ldots + 1 \cdot \delta^{\mu}_{\sigma} + \ldots + 0$$

$$= \delta^{\mu}_{\sigma}$$

(v) $$\frac{\partial \bar{x}^{\mu}}{\partial \bar{x}^{\nu}}\,\frac{\partial \bar{x}^{\nu}}{\partial \bar{x}^{\sigma}} = \frac{\partial \bar{x}^{\mu}}{\partial \bar{x}^{\sigma}} = \delta^{\mu}_{\sigma}$$

Scalars, Contravariant Vectors, and Covariant vectors.

Scalars:

Consider a function ϕ in a system of variables x^μ and let this function have the value $\bar{\phi}$ in another system of variables $\bar{x}^\mu$. Then if

$$\bar{\phi} = \phi$$

the function ϕ is said to be scalar or invariant or tensor of order zero. The quantity.

$$\delta^\mu_\mu = \delta^1_1 + \delta^2_2 + \cdots \delta^n_n = n$$

is a Scalar or an invariant.

Contravariant Vectors.

Consider a set of n quantities $A^1, A^2, A^3 \cdots A^n$ in a system of variables x^μ and these quantities have values $\bar{A}^1, \bar{A}^2, \bar{A}^3 \cdots \bar{A}^n$ in another system of variables $\bar{x}^\mu$. If these quantities obey the transformation law

$$\bar{A}^\mu = \frac{\partial \bar{x}^\mu}{\partial x^\alpha} A^\alpha$$

$$\left[\bar{A}^\mu = \sum_{\alpha=1}^{n} \frac{\partial \bar{x}^\mu}{\partial x^\alpha} A^\alpha\right]$$

then the quantities A^α are said to be the Components of a Contravariant Vector or a Contravariant tensor of first rank.

Covariant tensor.

Consider a set of n quantities $A_1, A_2, A_3 \cdots A_n$ in a system of variables x^μ and let these quantities have values $\bar{A}_1, \bar{A}_2, \bar{A}_3 \cdots \bar{A}_n$ in another system of variables $\bar{x}^\mu$. If these quantities obey transformation equations

$$\bar{A}_\mu = \frac{\partial x^\alpha}{\partial \bar{x}^\mu} A_\alpha$$

then the quantities A_α are said to be the Components of a Covariant vector or a Covariant tensor of rank one.

Tensors of Higher Ranks.

Contravariant tensors of Second Rank.

Let us Consider $(n)^2$ quantities $A^{\mu\nu}$ (here μ and ν take the values from 1 to n independent) in a System of variables x^μ and let these quantities have values $\bar{A}^{\mu\nu}$ in another System of variables $\bar{x}^\mu$. If these quantities obey the transformation equations

$$\bar{A}^{\mu\nu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} A^{\alpha\beta}$$

then the quantities $A^{\mu\nu}$ are said to be the Componen[t]

of a contravariant tensor of second rank

<u>covariant tensor of Second rank</u>.

If (n^2) quantities $A_{\mu\nu}$ in a system of variables x^{μ} are related to other (n^2) quantities in another system of variables $\bar{x}^{\mu}$ by the transformation equations

$$\bar{A}_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial \bar{x}^{\mu}} \frac{\partial x^{\beta}}{\partial \bar{x}^{\nu}} A_{\alpha\beta}$$

then the quantities $A_{\mu\nu}$ are said to be the Component of a covariant tensor of Second rank.

<u>Mixed tensors of Second Rank</u>.

If $(n)^2$ quantities A^{μ}_{ν} in a system of variables x^{μ} are related to the other $(n)^2$ quantities $\bar{A}^{\mu}_{\nu}$ in another system of variables $\bar{x}^{\mu}$ by the transformation equations

$$\bar{A}^{\mu}_{\nu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial \bar{x}^{\nu}} A^{\alpha}_{\beta}$$

then the quantities A^{μ}_{ν} are said to be the components of a mixed tensor of second rank.

Tensors of higher Ranks: Rank of a tensor

"The rank of a tensor only indicates the number of indicies attached to it per component."

For example $A^{\mu\nu\sigma}_{\lambda}$ are the components of a mixed tensor of rank 4: Contravariant of rank 3 and Covariant of rank 1: if they transform according to the equation

$$\bar{A}^{\mu\nu\sigma}_{\lambda} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial \bar{x}^{\sigma}}{\partial x^{\gamma}} \frac{\partial x^{\delta}}{\partial \bar{x}^{\lambda}} A^{\alpha\beta\gamma}_{\delta} \cdots$$

The rank of a tensor when raised as power to the number of dimensions gives the number of components of the tensor. For example a tensor of rank r in n-dimensional space has $(n)^r$ components. Thus the rank of a tensor gives the number of mode of changes of a physical quantity when passing from one system to the other which is in rotation relative to the first. Obviously, a quantity that remains unchanged when axes are rotated is a tensor of zero rank. The tensors of zero rank are Scalars or invariant and similarly the tensors of rank one are Vectors.

(iv) Contraction of tensors.

"The algebraic operation by which the rank of a mixed tensor is lowered by 2 is known as Contraction.

In the process of Contraction one Contravariant index and one Covariant index of a mixed tensor are set equal and the repeated index is summed over, the result is a tensor of rank lower by two than the original tensor".

For ex: Consider a mixed tensor $A_{\lambda\rho}^{\mu\nu\sigma}$ of rank 5 with Contravariant indices $\mu\nu\sigma$ and Covariant indices λ,ρ

The transformation law for the given tensor is

$$\bar{A}_{\lambda\rho}^{\mu\nu\sigma} = \frac{\partial\bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial\bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial\bar{x}^{\sigma}}{\partial x^{\gamma}} \frac{\partial x^{\delta}}{\partial\bar{x}^{\lambda}} \frac{\partial x^{\tau}}{\partial\bar{x}^{\rho}} A_{\delta\tau}^{\alpha\beta\gamma} \quad \text{---} (12)$$

To apply the process of Contraction we put $\rho=\sigma$ and obtain

$$\bar{A}_{\lambda\sigma}^{\mu\nu\sigma} = \frac{\partial\bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial\bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial\bar{x}^{\sigma}}{\partial x^{\gamma}} \frac{\partial x^{\delta}}{\partial\bar{x}^{\lambda}} \frac{\partial x^{\tau}}{\partial\bar{x}^{\sigma}} A_{\delta\tau}^{\alpha\beta\gamma}$$

$$= \frac{\partial\bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial\bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial x^{\delta}}{\partial\bar{x}^{\lambda}} \frac{\partial x^{\tau}}{\partial\bar{x}^{\sigma}} \frac{\partial\bar{x}^{\sigma}}{\partial x^{\gamma}} A_{\delta\tau}^{\alpha\beta\gamma}$$

$$= \frac{\partial\bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial\bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial x^{\delta}}{\partial\bar{x}^{\lambda}} \delta_{\gamma}^{\tau} A_{\delta\tau}^{\alpha\beta\gamma}$$

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$$\left(\text{Since } \frac{\partial x^{\tau}}{\partial \bar{x}^{\sigma}} \frac{\partial \bar{x}^{\sigma}}{\partial x^{\gamma}} = \frac{\partial x^{\tau}}{\partial x^{\gamma}} = \delta_{\gamma}^{\tau}\right)$$

$$= \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial x^{\delta}}{\partial \bar{x}^{\lambda}} A_{\delta\gamma}^{\alpha\beta\gamma} \quad \ldots (13)$$

$$\bar{A}_{\lambda}^{\mu\nu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{\nu}}{\partial x^{\beta}} \frac{\partial x^{\delta}}{\partial \bar{x}^{\lambda}} A_{\delta}^{\alpha\beta}$$

which is a transformation law for a mixed tensor of rank 3. Hence $A_{\lambda\sigma}^{\mu\nu\sigma}$ is a mixed tenser of rank 3 and may be denoted by $A_{\delta}^{\alpha\beta}$. In this example we can further apply the contraction process $\lambda = \nu$ and obtain the contravariant vector $A_{\nu\sigma}^{\mu\nu\sigma}$ or A^{μ}

Ex: Consider the contraction of the mixed tensor A_{ν}^{μ} of rank 2, whose transformation law is

$$\bar{A}_{\nu}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial \bar{x}^{\nu}} A_{\beta}^{\alpha}$$

To apply contraction process we put $\nu = \mu$ and obtain

$$\bar{A}_{\mu}^{\mu} = \frac{\partial \bar{x}^{\mu}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial \bar{x}^{\mu}} A_{\beta}^{\alpha} = \delta_{\alpha}^{\beta} A_{\beta}^{\alpha} = A_{\alpha}^{\alpha} = A$$

A_{ν}^{μ} is justified to call a scalar a tensor of rank zero.

Symmetric and Antisymmetric tensors

a) Symmetric tensors

"If two Contravariant or Covariant indices can be interchanged without altering the tensor, then the tensor is said to be symmetric with respect to these two indices".

For example if

$$\left.\begin{aligned} A^{\mu\nu} &= A^{\nu\mu} \\ A_{\mu\nu} &= A_{\nu\mu} \end{aligned}\right\} \quad (1)$$

then the Contravariant tensor of second rank $A^{\mu\nu}$ or Covariant tensor of second rank $A_{\mu\nu}$ is said to be symmetric.

For a tensor of higher rank $A_{\lambda}^{\nu\mu\sigma}$ if

$$A_{\lambda}^{\mu\nu\sigma} = A_{\lambda}^{\nu\mu\sigma}$$

then the tensor $A_{\lambda}^{\mu\nu\sigma}$ is said to be symmetric with respect to indices μ and ν.

The symmetry property of a tensor is independent of coordinate system used. So if a tensor is symmetric with respect to two indices in any coordinate system, it remains symmetric with respect to these two indices in any other coordinate system. This can be seen as follows.

If tensor $A_\lambda^{\mu\nu\sigma}$ is symmetric with respect to first two indices μ and ν, we have

$$A_\lambda^{\mu\nu\sigma} = A_\lambda^{\nu\mu\sigma}$$

According to tensor transformation law

$$\bar{A}_\lambda^{\mu\nu\sigma} = \frac{\partial \bar{x}^\mu}{\partial x^\alpha} \frac{\partial \bar{x}^\nu}{\partial x^\beta} \frac{\partial \bar{x}^\sigma}{\partial x^\gamma} \frac{\partial x^\delta}{\partial \bar{x}^\lambda} A_\delta^{\alpha\beta\gamma} \quad --(1)$$

Now interchanging the dummy indices α and β

we get $$\bar{A}_\lambda^{\mu\nu\sigma} = \frac{\partial \bar{x}^\mu}{\partial x^\beta} \frac{\partial \bar{x}^\nu}{\partial x^\alpha} \frac{\partial \bar{x}^\sigma}{\partial x^\gamma} \frac{\partial x^\delta}{\partial \bar{x}^\lambda} A_\delta^{\alpha\beta\gamma}$$

$$= \frac{\partial \bar{x}^\nu}{\partial x^\alpha} \frac{\partial \bar{x}^\mu}{\partial x^\beta} \frac{\partial \bar{x}^\sigma}{\partial x^\gamma} \frac{\partial x^\delta}{\partial \bar{x}^\lambda} A_\delta^{\alpha\beta\gamma}$$

$$= \bar{A}_\lambda^{\nu\mu\sigma}$$

ie given tensor is again symmetric with respect to first two indices in the new coordinate system. This result can also be proved for covariant indices. Thus the symmetry property of a tensor is independent of coordinate system.

We cannot usually define symmetry property with respect to two indices, of which ~~are~~ one contravariant and the other covariant, because this symmetry property cannot be preserved after a coordinate transformation. It

b) Antisymmetric Tensors.

* A Tensor, whose each Component alters in sign but not in magnitude, when two Contravariant or Covariant indices are interchanged, is said to be skew-symmetric or antisymmetric with respect to these two indices. For example if

$$A^{\mu\nu} = -A^{\nu\mu}$$

$$A_{\mu\nu} = -A_{\nu\mu}$$

The contravariant tensor $A^{\mu\nu}$ or Covariant tensor $A_{\mu\nu}$ of second rank is antisymmetric.

For a tensor of higher rank $A_{\lambda}^{\mu\nu\sigma}$, if

$$A_{\lambda}^{\mu\nu\sigma} = -A_{\lambda}^{\mu\sigma\nu}$$

The tensor $A_{\lambda}^{\mu\nu\sigma}$ is antisymmetric with respect to indices ν and σ.

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The skew symmetric property of a tensor is also independent of the ~~Inde~~ choice of coordinate system. So if a tensor is skew-symmetric with respect to two indices in any other coordinate system. This can be seen as follows.

If a tensor $A_\lambda^{\mu\nu\sigma}$ is antisymmetric with respect to first two indices μ and ν, then we have

$$A_\lambda^{\mu\nu\sigma} = -A_\lambda^{\nu\mu\sigma}$$

By tensor transformation law, we have

$$\bar{A}_\lambda^{\mu\nu\sigma} = \frac{\partial \bar{x}^\mu}{\partial x^\alpha} \frac{\partial \bar{x}^\nu}{\partial x^\beta} \frac{\partial \bar{x}^\sigma}{\partial x^\gamma} \frac{\partial x^\delta}{\partial \bar{x}^\lambda} A_\delta^{\alpha\beta\gamma}$$

Now interchanging dummy indices α and β. we get

$$\bar{A}_\lambda^{\mu\nu\sigma} = -\frac{\partial \bar{x}^\mu}{\partial x^\beta} \frac{\partial \bar{x}^\nu}{\partial x^\alpha} \frac{\partial \bar{x}^\sigma}{\partial x^\gamma} \frac{\partial x^\delta}{\partial \bar{x}^\lambda} A_\delta^{\beta\alpha\gamma}$$

$$= -\frac{\partial \bar{x}^\nu}{\partial x^\alpha} \frac{\partial \bar{x}^\mu}{\partial x^\beta} \frac{\partial \bar{x}^\sigma}{\partial x^\gamma} \frac{\partial x^\delta}{\partial \bar{x}^\lambda} A_\delta^{\alpha\beta\gamma}$$

$$= -\bar{A}_\lambda^{\nu\mu\sigma}$$

i.e given tensor is again antisymmetric with respect to first to indices in new Coordinate System.