5.1. Introduction

X-rays were discovered by Roentgen in 1895. X-rays are electromagnetic waves of short wavelengths in the range of 10 Å to 0.5 Å. The longer wavelength end of the spectrum is known as the "*soft* X-rays" and the shorter wavelength end is known as "*hard* X-rays."

5.2. Production of X-rays

The Coolidge tube. X-rays are produced when fast moving electrons are suddenly stopped by a solid target. A Coolidge tube is shown in Fig. 5.1.
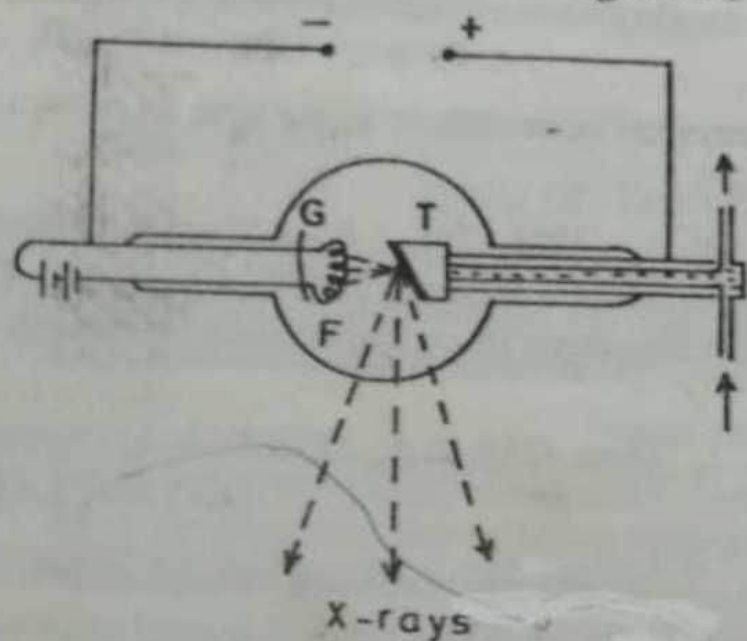

X-rays

Fig. 5.1

The tube is exhausted to the best possible vacuum of the order of 10^{-5} mm of mercury. The cathode consists of a tungsten filament (F) heated by a low tension battery. Thermionic electrons emitted by the filament are accelerated towards the target (T) by a high P.D. maintained between F and T. The filament is placed inside a metal cup G to focus the electrons on to the target.

The target must be cooled to remove the heat generated in it by continuous electron-bombardment. The usual method is to mount the target material on a hollow copper tube through which cold water is continuously circulated. The target is made of a metal like tungsten or molybdenum having a high melting point and a high atomic number. Metals with high atomic number give more energetic and intense X-rays when used as targets.

ily controlled.

(1) The intensity of X-rays depends on the number of electrons striking the target per second. The number of electrons given out by the filament is proportional to its temperature, which can be adjusted by varying the current in the filament circuit. Therefore, the intensity of X-rays varies with the filament current.

(2) The frequency of X-rays emitted depends on the voltage between the cathode and the anode (target). Let V be the accelerating potential across the tube. If e is the charge on the electron, the work done on the electron in moving from the cathode to the anticathode $= eV$. The electron thus acquires K.E. which is converted into X-rays, when the electron strikes the target. If v_{max} is the maximum frequency of the X-rays produced, then $h\, v_{max} = eV$.

∴ The minimum wavelength produced by an X-ray tube
$$\left. \right\} = \lambda_{min} = \frac{c}{v_{max}} = \frac{hc}{eV}.$$

5.3 Spacing between Three Dimensional Lattice Planes

Consider O as the origin and OX, OY, OZ as three rectangular axes (Fig. 5.1a). Let a reference plane pass through O, and consider a set of parallel planes defined by Millerindices (hkl). Beside reference plane through O, if next plane passes through A, B and C then the respective intercepts are $a/h, b/k$ and c/l. If ON is the normal between this plane and the reference plane, then $ON = d$ is the interplanar spacing.
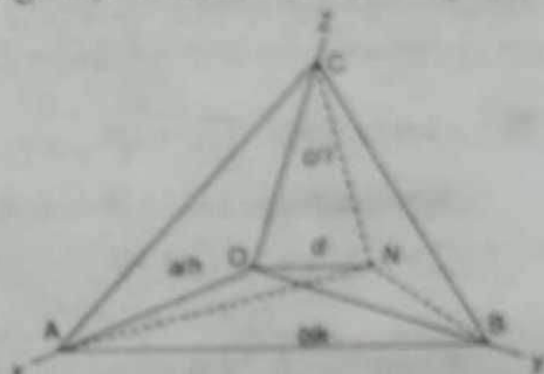
Let us now find the value of d in terms of a, b and c. Let the normal make angles α, β, γ with crystal axes such that $\angle NOX = \alpha$, $\angle NOY = \beta$, and $\angle NOZ = \gamma$.

We have from Fig. 5.1a,

Fig. 5.1a

$$d = \frac{a}{h}\cos\alpha = \frac{b}{k}\cos\beta = \frac{c}{l}\cos\gamma$$

or
$$\cos\alpha = \frac{d}{(a/h)}, \cos\beta = \frac{d}{(b/k)}, \cos\gamma = \frac{d}{(c/l)}$$

But
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

∴
$$\left[\frac{d}{(a/h)}\right]^2 + \left[\frac{d}{(b/k)}\right]^2 + \left[\frac{d}{(c/l)}\right]^2 = 1$$

or
$$d^2\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right] = 1$$

or
$$d = \left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right]^{-\frac{1}{2}}$$

Derivation of Bragg's law. Consider a set of parallel planes of atom points at a spacing d between two successive planes. Let a narrow mono-
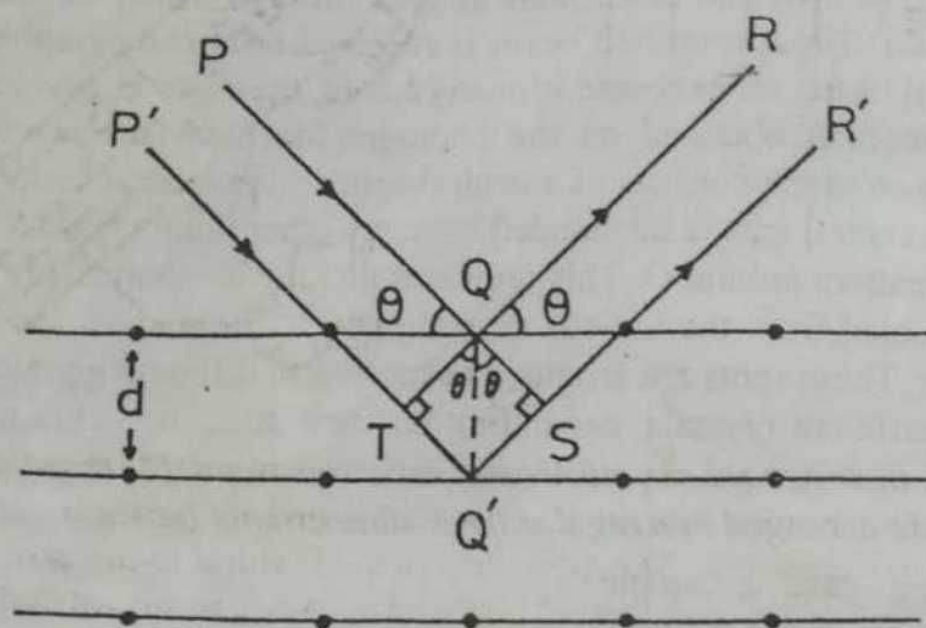

Fig. 5.6

chromatic X–ray beam of wavelength λ be incident on the first plane at a glancing angle θ (Fig. 5.6). Consider the ray PQ incident on the first plane. The corresponding reflected ray QR must also be inclined at the same angle θ to the plane. Since X-rays are much more penetrating than ordinary light, there is only partial reflection at each plane. The complete absorption takes place only after penetrating several layers. Consider two parallel rays PQR and $P'Q'R'$ in the beam, which are reflected by two atoms Q and Q'. Q' is vertically below Q. The ray $P'Q'R'$ has a longer path than the ray PQR. To compute the path-difference between the two rays, from Q draw normals QT and QS on $P'Q'$ and $Q'R'$ respectively. Then

the path-difference $= TQ' + Q'S = d \sin \theta + d \sin \theta = 2d \sin \theta$.

Hence the two rays will reinforce each other and produce maximum intensity, if

$$2d \sin \theta = n \lambda,$$

where $n = 1, 2, 3......$ The integer n gives the order of the scattered beam, λ is the wavelength of the X-rays used. This equation is called *Bragg's law.*

5.7. The Bragg X-ray spectrometer

The essential parts of a Bragg spectrometer are shown in Fig. 5.7. It is similar in construction to an optical spectrometer. It consists of three parts. (1) a source of X-rays (2) a crystal held on a circular table which is graduated and provided with vernier and (3) a detector (ionisation chamber). X-rays from an X-ray tube, limited by two narrow lead slits S_1 and S_2 ,are
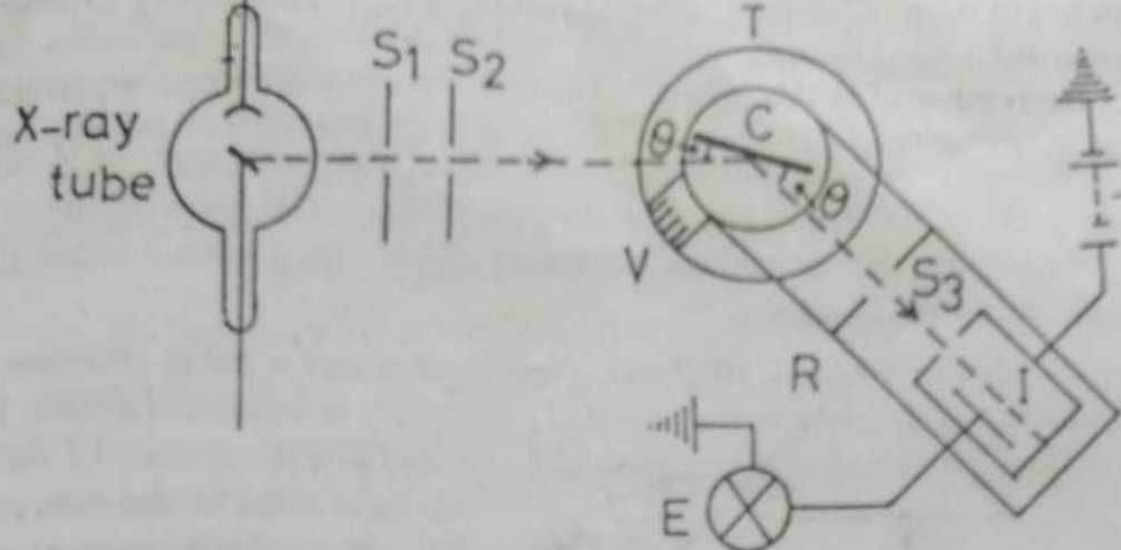
Fig. 5.7

allowed to fall upon the crystal C. The crystal is mounted on the circular table T, which can rotate about a vertical axis and its position can be determined by the vernier V. The table is provided with a radial arm (R) which carries an ionisation chamber (I). This arm also can be rotated about the same vertical axis as the crystal. The position of this arm can be determined by a second vernier (not shown in the figure). The ionisation chamber is connected to an electrometer (E) to measure the ionisation current. Hence we can measure the intensity of the diffracted beam of X-rays, diffracted in the direction of the ionisation chamber. S_3 is a lead slit, to limit the width of the diffracted beam. In practice, the crystal table is geared to the ionisation chamber so that the chamber turns through 2θ when the crystal is turned through θ.

Working. To begin with, the glancing angle θ for the incident beam is kept very small. The ionisation chamber is adjusted to receive the reflected beam ti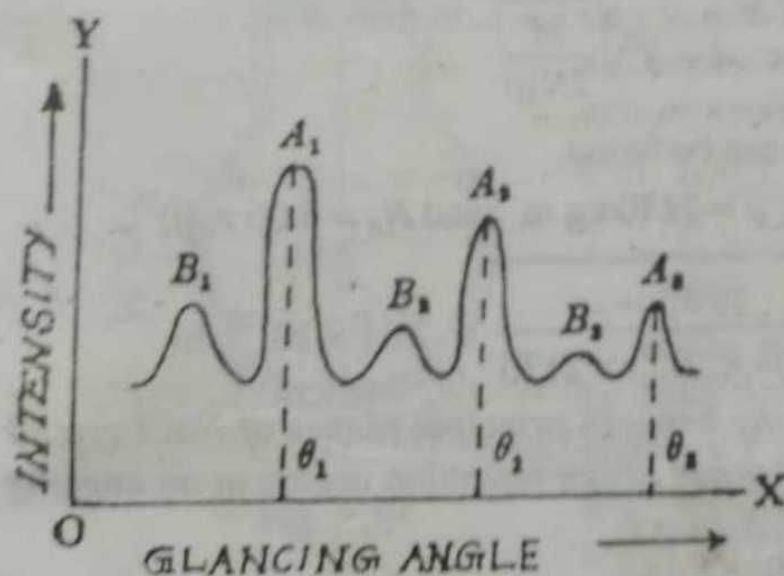ll the rate of deflection is a maximum. The glancing angle (θ) and the intensity of the diffracted beam (I) are measured. The glancing angle is next increased in equal steps, by rotating the crystal table. The ionisation current is noted for different glancing angles. The graph of ionisation current against glancing angle is drawn. The graph obtained is as in Fig. 5-8 and is called an X-ray spectrum. The prominent peaks A_1, A_2, A_3 refer to X-rays of wavelength λ. The glancing angles θ_1, θ_2, θ_3 corresponding to the peaks A_1, A_2, A_3 are obtained from the graph. It is found that $\sin\theta_1 : \sin\theta_2 : \sin\theta_3 = 1 : 2 : 3$. This shows that A_1, A_2, A_3, refer to the first, second and third order reflections of the same wavelength. B_1, B_2, B_3 are such peaks for the first

Fig. 5.8

second and third order for another wavelength (λ_2). Thus Bragg experimentally verified the relation

$$2d \sin \theta = n\lambda.$$

Measurement of wavelength of X-rays. The wavelength of X-rays is determined by using the equation $2d \sin\theta = n\lambda$. The glancing along θ is experimentally determined as explained already for a known order. If d is known, λ can be calculated.

Calculation of d. Rocksalt (NaCl) possesses a cubic structure with sodium ions and chlorine ions situated alternately at corners of a cube. If d is the distance between two n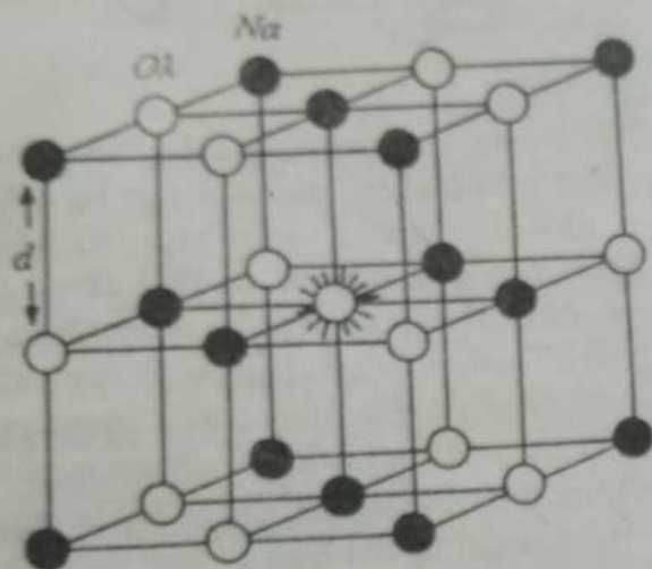eighbouring ions and ρ is the density of the crystal, then mass of the unit cube = ρd^3. Now each corner ion is shared by 8 neighbour cubes. This is illustrated by the ion indicated by an asterisk in Fig. 5.9. Hence each ion contributes only $\frac{1}{8}$ th of its mass to the cube. The unit cube is made up of 4 sodium ions and 4 chlorine ions. Hence, the mass of each cube = the mass of $\frac{1}{2}$ NaCl molecule. If M is the molecular weight of NaCl and N_A the Avogardo's No.,

Fig. 5.9

Mass of $\frac{1}{2}$ NaCl molecule $= M/2N_A$

$$\therefore \quad \rho d^3 = \frac{M}{2N_A} \quad \text{or} \quad d = 3\sqrt{\frac{M}{2N_A\rho}}$$

Knowing M, N_A and ρ, d can be found.
For NaCl, $M = 58.45$ kg; $\rho = 2170$ kg m^{-3} and $N_A = 6.06 \times 10^{26}$.

$$\therefore \quad d = 3\sqrt{\frac{58.45}{2 \times (6.06 \times 10^{26}) \times 2170}} = 2.81 \times 10^{-10} \text{ m}.$$

Example 1. The spacing between principal planes of NaCl crystal is 2.82 Å. It is found that first order Bragg reflection occurs at an angle of 10°. What is the wavelength of X-rays?

By Bragg equation, $2d \sin\theta = n\lambda$.
Here, $d = 2.82 \times 10^{-10}$ m; $n = 1$ and $\theta = 10°$. $\lambda = ?$

$$\lambda = \frac{2d \sin\theta}{n} = \frac{2 \times (2.82 \times 10^{-10}) \sin 10°}{1} = 0.98 \times 10^{-10} \text{ m}.$$

Example 2. Bragg's spectrometer is set for the first order reflection

One has hexagonal symmetry and is called the hexagonal close-packed structure. The other has cubic symmetry and is called the face-centred cubic structure.

Spheres may be arranged in a single closest-packed layer by placing each sphere in contact with six others (Fig. 5.14 a). The centres of the spheres are at points marked A. A second and identical layer of spheres can be placed over this, with centres over the points marked B (or equivalently C). If the second layer goes in over B, there are two non-equivalent choices for a third layer. It can go in over A or over C. If the third layer goes in over A, the sequence is ABABAB..... and the struc-
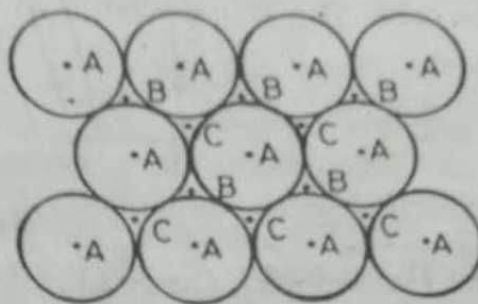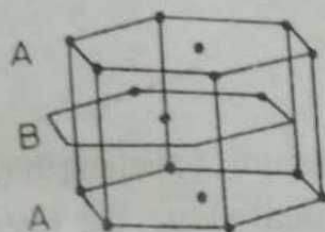

Fig. 5.14 (a)

ture is hcp (Fig. 5.14b).Examples of elements which have this structure are Cd, Mg, Ti, Zn, etc.

If the third layer goes in over C, the sequence is ABCABCABC and the structure is fcc (Fig. 5.14c). Elements having fcc structure are Cu, Ag, Au, Al, Pd, Pt, etc.

In either structure, each atom has 12 nearest neighbours. So the *coordination number* is 12. The packing fraction is 0.74 for both the fcc and hcp structures.

Fig. 5.14 (b)

The Cesium Chloride Structure.
The structure of CsCl is shown in Fig 5.14d. The Cs^+ ions form an sc lattice interlocked with another sc lattice formed by the Cl^- ions. There is one molecule per primitive cell with ions at the corners 000 and body centred positions $\frac{1}{2}\frac{1}{2}\frac{1}{2}$ of the simple cubic space lattice. Each ion is at the centre of a cube of ions of the opposite kind, so that the coordination number is 8. TlBr, TlI, NH4Cl, CuPd, CuZn (β-brass), AgMg, LiHg, AlNi, BeCu are representative crystals having the CsCl arrangement.
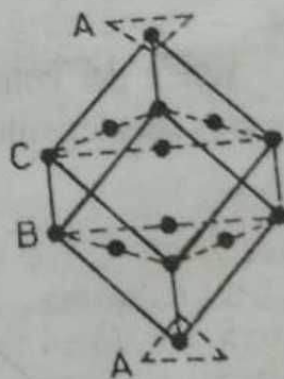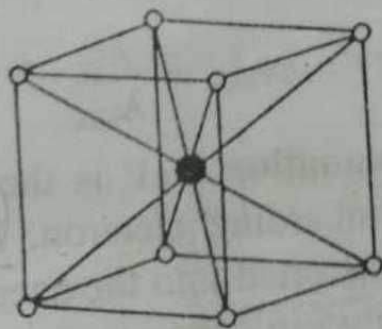

Fig. 5.14 (c)

5.11 X-ray spectra
Urey and his co-workers analysed the X-ray beam emitted from an X-ray tube using different potential differences and same target. Using tungsten as the target and different potential differences, the intensities of the rays produced are plotted against wavelength. Fig. 5.15 represents these
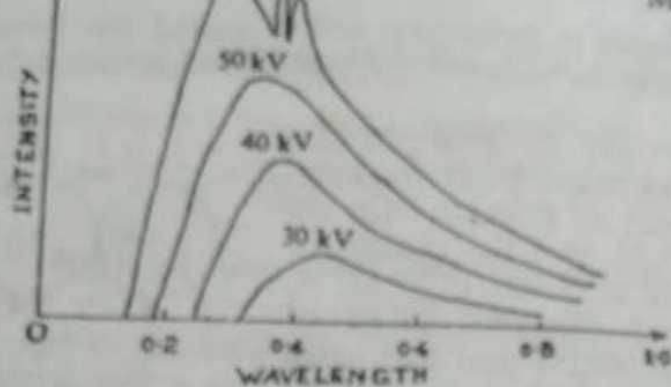

o Cs ions
• Cl ion

Fig. 5.14 (d)

Fig. 5.15

graphs. For applied P.D.s 30 kV, 40 kV and 50kV, the spectrum is *white*. But, for the applied P.D. 70 kV, two sharp peaks are seen. The sharp peaks show the *line* or *characteristic radiation*. The line spectra are absent, till the P.D. is greater than a particular value. The smoothly varying curves represent the *continuous spectrum*. The superimposed lines on the continuous background constitute the *characteristic spectrum*.

Main features of continuous X-ray spectrum

(1) For each anode potential, there is a minimum wavelength (λ_{min}) below which no radiation is emitted. Above this critical value, the intensity of the radiation increases rapidly with increasing wavelengths and after reaching a maximum, decreases gradually. The intensity never reaches zero showing that the radiation contains all possible wavelengths above the minimum limit.

(2) When the voltage across the X-ray tube is increased, λ_{min} is shifted towards smaller values. Duane and Hunt showed that λ_{min} is inversely proportional to the applied voltage V or v_{max} is directly proportional to V. If the limiting frequencies (v_{max}) are plotted against the applied voltages (V), a straight line graph passing through the origin is obtained (Fig. 5.16). This empirical law of Duane and Hunt is expressed analytically as

Fig. 5.16

$$eV = hv_{max} = \frac{hc}{\lambda_{min}}$$

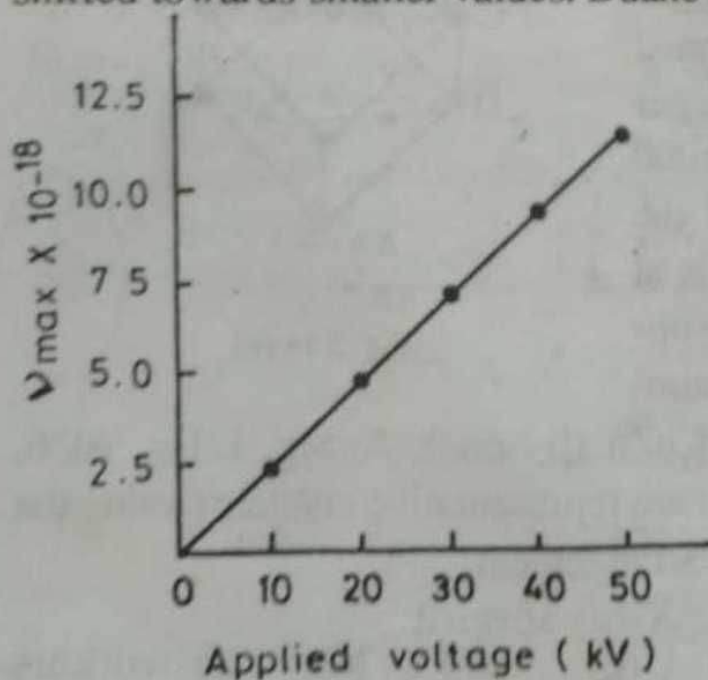**Explanation.** eV is the K.E. of the bombarding electron. If the entire K.E. of the electron striking the target is converted into the energy of the X-ray photon, then, $eV = hv_{max}$ according to Einstein's theory.

But
$$V_{max} = \frac{c}{\lambda_{min}}$$

$$\therefore \quad eV = \frac{hc}{\lambda_{min}} \quad \text{or} \quad \lambda_{min} = \frac{hc}{eV} \quad \text{or} \quad \lambda_{min} \propto \frac{1}{V}$$

Most of the electrons that generate X-ray photons give up only a part of their energy in this way. Therefore, most of the X-radiation is of longer wavelength than λ_{min}. Thus, the continuous spectrum is the result of the inverse photoelectric effect, with electron kinetic energy (eV) being transformed into photon energy (hv).

5.12. Characteristic X-ray spectrum

There are two methods of producing characteristic X-rays.

(1) The characteristic X-rays of an element can be excited by using the element as the target in the X-ray tube and thus subjecting it to direct bombardment by electrons. For each target there is a minimum potential below which the line spectra do not appear. This critical P.D. below which the line spectra do not appear, is different for different targets. Molybdenum shows up the line spectra only if the P.D. is above 35 kV.

(2) Characteristic X-rays of an element can also be excited by allowing primary X-rays from a hard X-ray tube to fall on the element. The primary X-rays must be harder than the characteristic X-rays to be produced. The peaks obtained in the X-ray spectrum (Fig. 5.15) give us the line spectrum which is characteristic of the element used in the target. The group of lines of shortest wavelength is called the K-series. Usually two lines of this series are detected. These lines are termed as K_α and K_β lines in the order of decreasing wavelengths. The next group is called the L-series of
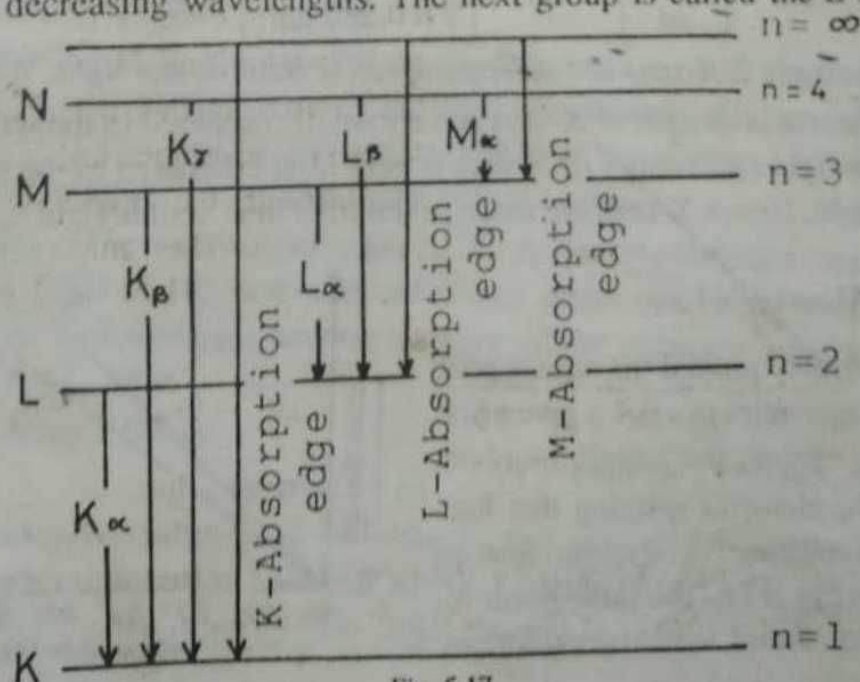

Fig. 5.17

longer wavelengths (L_α, L_β, L_γ etc.). For heavier elements a third series, called the M-series has been detected.

Origin of characteristic X-rays. This can be understood in terms of Bohr's theory. Suppose an atom in the target of an X-ray tube is bombarded by a high-speed electron and a K-electron is removed. A vacancy is created in the K-shell. This vacancy can be filled up by an electron from either of L, M or N shells or a free electron. These possible transitions can result in the K_α, K_β lines and the limiting line. Similarly, the longer wavelength L-series originates when an L electron is knocked out of the atom, the M-series when an M electron is knocked out and so on (Fig. 5.17).

Note. It is clear that continuous spectra and line spectra both emitted by the same target are of different origin. The continuous spectrum is the result of the inverse photoelectric effect, with electron K.E. being transformed into photon energy hv. The line spectrum has its origin in electronic transitions within atoms that have been disturbed by the incident electrons.

Example 1. *An X-ray tube with Cu target is operated at 28 kV. The smallest glancing angle for NaCl crystal for the Cu K_α line is 15.8°. Find the wavelength of this line. Also find the glancing angle for photons at the short wavelength limit. (d for NaCl = 0.282 nm).*

Solution. From Bragg's relation, $2d \sin \theta = n\lambda$. Here, $n = 1$

$\therefore \quad \lambda = 2d \sin \theta = 2 \times (0.282 \text{ nm}) \times \sin 15.8° = 0.153 \text{ nm}$

$$\lambda_{min} = \frac{hc}{eV} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 28 \times 10^3} \text{ m} = 0.0496 \text{ nm}$$

Let θ' be the glancing angle corresponding to λ_{min}. Then

$$\theta' = \sin^{-1} \left(\frac{\lambda_{min}}{2d} \right) = \sin^{-1} \left(\frac{0.0496 \text{ nm}}{2 \times 0.282 \text{ nm}} \right) = 5.1°$$

Example 2. *X-rays are more penetrative than visible light. Why?*

Ans. Wavelength of X-rays is very small compared to the wavelength of visible light *i.e.*, energy of X-rays is very high compared to the energy of visible light. Hence X-rays are more penetrative than visible light.

5.13. Moseley's Law

Moseley plotted the square root of the frequencies ($\sqrt{v}$) of a given line (say K_α) against the atomic numbers (Z) of the elements emitting that line. Moseley obtained a straight line as shown in Fig. 5.18. The same linear relation was found to hold good for any
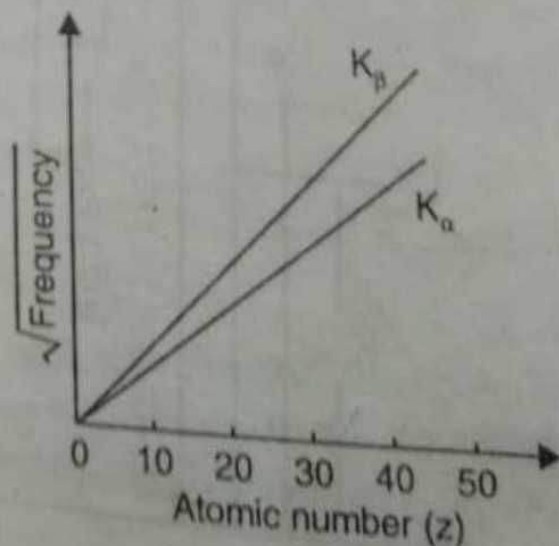

Fig. 5.18

line in any series. He concluded, therefore, that atomic number (and not atomic weight) is the fundamental property of elements.

Moseley's law: statement. *The frequency of a spectral line in X-ray spectrum, varies as the square of the atomic number of the element emitting it, or* $v \propto Z^2$. Moseley's law may be written as $\sqrt{v} = a(Z-b)$. Here, Z is the atomic number of the element and a and b are constants depending upon the particular line.

Explanation, according to Bohr's theory. Bohr's theory of hydrogen spectrum gives the frequency of a spectral line as $v = Z^2 Rc \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2} \right)$

where R is Rydberg's constant and c the velocity of light. Considering the K_α line, we can regard it as originating from the transition of electron from second to first orbit. Now, $n_1 = 1$ and $n_2 = 2$.

So frequency of K_α line $= v = Z^2 Rc \left(\dfrac{1}{1^2} - \dfrac{1}{2^2} \right) = \dfrac{3}{4} cRZ^2$

This approximately corresponds to Moseley's law.

Importance of Moseley's law. (1) According to this law, it is the atomic number and not atomic weight of an element which determines its characteristic properties, both physical and chemical. Therefore, the atoms must be arranged in the periodic table according to their atomic numbers and not according to their atomic weights. This would remove some discrepancies in the order of certain elements from the point of view of their atomic weights. For example, argon $_{18}Ar^{40}$ comes before potassium ($_{19}K^{39}$), cobalt ($_{27}Co^{58.9}$) comes before nickel ($_{28}Ni^{58.7}$) etc. So the arrangement is correct in the order of atomic number.

(2) Moseley's work has also helped to perfect the periodic table by (i) the discovery of new elements, e.g., hafnium (72), illinium (61), masurium (43), rhenium (75), etc., and (ii) the determination of the atomic numbers of rare-earths and fixing their positions in the periodic table.

Example. *The wavelength of the L_α X-ray line of plantinum (atomic number 78) is 1.321 Å. An unknown substance emits L_α X-rays of wavelength 4.174 Å. Calculate the atomic number of the unknown substance Given $b = 7.4$ for L_α lines.*

Sol. Moseley's law is

$$\sqrt{v} = a(z - b)$$

Let v_1 and v_2 be the frequencies of the L_α line of platinum and the unknown substance respectively. Let their atomic numbers be z_1 and z_2 respectively. Then

$$\sqrt{v_1} = a(z_1 - b) \quad \text{and} \quad \sqrt{v_2} = a(z_2 - b)$$

or $\qquad \sqrt{\dfrac{v_1}{v_2}} = \dfrac{z_1-b}{z_2-b}$ or $z_2 - b = (z_1-b)\sqrt{\dfrac{v_1}{v_1}}$

Since $c = v\lambda$, we have $v_2/v_1 = \lambda_1/\lambda_2$

$\therefore \quad z_2 - b = (z_1 - b)\sqrt{\dfrac{\lambda_1}{\lambda_2}}$ or $z_2 = b + (z_1 - b)\sqrt{\dfrac{\lambda_1}{\lambda_2}}$

Here, $b = 7.4$; $z_1 = 78$; $\lambda_1 = 1.321$ Å and $\lambda_2 = 4.174$ Å

$\therefore \quad z_2 = 7.4 + (78 - 7.4)\sqrt{1.321/4.174} = 47.12$

The atomic number of the unknown substance is 47.12.

5.14. Compton Scattering

The Compton Effect. *Compton discovered that when X-rays of a sharply defined frequency were incident on a material of low atomic number like carbon, they suffered a change of frequency on scattering. The scattered beam contains two wavelengths. In addition to the expected incident wavelength, there exists a line of longer wavelength. The change of wavelength is due to loss of energy of the incident X-rays. This elastic interaction is known as Compton effect.*

In the case of incoherent scattering, a scattered beam undergoes not only deviation in its direction but also change of wavelength occurs. In Compton effect, there is a change in wavelength of the scattered beam along with the change in its direction. Hence Compton effect is an incoherent scattering.

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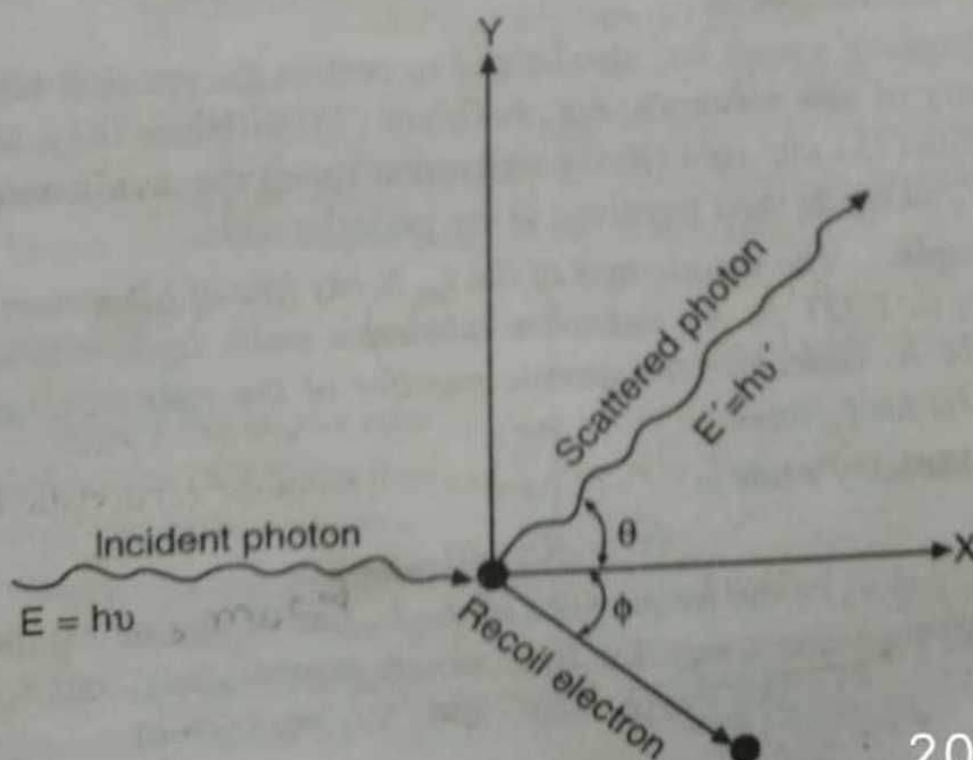
This effect was explained by Compton on the basis of quantum theory of radiation. The whole process is treated as a particle collision event between X-ray photon and a loosely bound electron of the scatterer. In this process, both momentum and energy are conserved. In the photon-electron collision, a portion of the energy of the photon is transferred to the electron. As a result, the X-ray proceeds with less than the original energy (and therefore has a lower frequency or a higher wavelength).

The incident photon with an energy $h\nu$ and momentum $h\nu/c$ strikes an electron at rest. The initial momentum of the electron is zero and its initial energy is only the rest mass energy, m_0c^2. The scattered photon of energy $h\nu'$ and momentum $h\nu'/c$ moves off in a direction inclined at an angle θ to the original direction. The electron acquires a momentum mv and moves at an angle ϕ to the original direction. The energy of the recoil electron is mc^2 (Fig. 5.19).

According to the principle of conservation of energy,

$$h\nu + m_0c^2 = h\nu' + mc^2 \qquad \text{...(1)}$$

Considering the x and y components of the momentum and applying the principle of conservation of momentum,

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + mv\cos\phi \qquad \text{...(2)}$$

and

$$0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \qquad \text{...(3)}$$

From (2), $mvc\cos\phi = h(\nu - \nu'\cos\theta)$ ...(4)

From (3), $mvc\sin\phi = h\nu'\sin\theta$ ...(5)

Squaring and adding (4) and (5),

$$m^2v^2c^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2\cos^2\theta) + h^2\nu'^2\sin^2\theta$$
$$= h^2(\nu^2 - 2\nu\nu'\cos\theta) + h^2\nu'^2 = h^2(\nu^2 - 2\nu\nu'\cos\theta + \nu'^2) \quad \text{...(6)}$$

From (1), $mc^2 = h(\nu - \nu') + m_0c^2$

$$m^2c^4 = h^2(\nu^2 - 2\nu\nu' + \nu'^2) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 \qquad \text{...(7)}$$

Subtracting (6) from (7),

$$m^2c^2(c^2 - v^2) = -2h^2\nu\nu'(1 - \cos\theta) + 2h(\nu - \nu')m_0c^2 + m_0^2c^4 \qquad \text{...(8)}$$

The value of $m^2c^2(c^2 - v^2)$ can be obtained from the relativistic formula

$$m = \frac{m_0}{\sqrt{(1 - v^2/c^2)}}. \text{ Squaring,}$$

$$m^2 = \frac{m_0^2}{1 - v^2/c^2} = \frac{m_0^2c^2}{c^2 - v^2}$$

$$\therefore \quad m^2c^2(c^2 - v^2) = m_0^2c^4 \qquad \text{....(9)}$$

From (8) and (9),

$$m_0^2 c^4 = -2h^2 vv'(1 - \cos \theta) + 2h(v - v')m_0 c^2 + m_0^2 c^4$$

$$\therefore \quad 2h(v - v')m_0 c^2 = 2h^2 vv'(1 - \cos \theta)$$

or $\dfrac{v - v'}{vv'} = \dfrac{h}{m_0 c^2}(1 - \cos \theta)$ or $\dfrac{1}{v'} - \dfrac{1}{v} = \dfrac{h}{m_0 c^2}(1 - \cos \theta)$

or $\dfrac{c}{v'} - \dfrac{c}{v} = \dfrac{h}{m_0 c}(1 - \cos \theta)$ or $\lambda' - \lambda = \dfrac{h}{m_0 c}(1 - \cos \theta)$ ...(10)

$\therefore$ The change in wavelength $= d\lambda = \dfrac{h}{m_0 c}(1 - \cos \theta)$

This relation shows that $d\lambda$ is independent of the wavelength of the incident radiations as well as the nature of the scattering substance. $d\lambda$ depends upon the angle of scattering only.

Case 1. When $\theta = 0$, $\cos \theta = 1$ and hence $d\lambda = 0$

Case 2. When $\theta = 90°$, $\cos \theta = 0$ and hence

$$d\lambda = \frac{h}{m_0 c} = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31}) \times (3 \times 10^8)} m = 0.0243\,\text{Å}$$

This is known as *Compton wavelength*.

Case 3. When $\theta = 180°$, $\cos \theta = -1$ and hence $d\lambda = 2h/m_0 c = 0.0485$ Å. $d\lambda$ has the maximum value at $\theta = 180°$.

Experimental verification. Monochromatic X-rays of wavelength λ are allowed to fall on a scattering material like a small block of carbon (Fig. 5.20). The scattered X-rays are received by a Bragg spectrometer and their wavelength is determined. The spectrometer can freely swing in an arc about the scatterer. The wavelength of the scattered X-rays is measured for different values of the scattering angle. The experimental results obtained by Compton are shown in Fig. 5.21. In the scattered radiation in addition to the
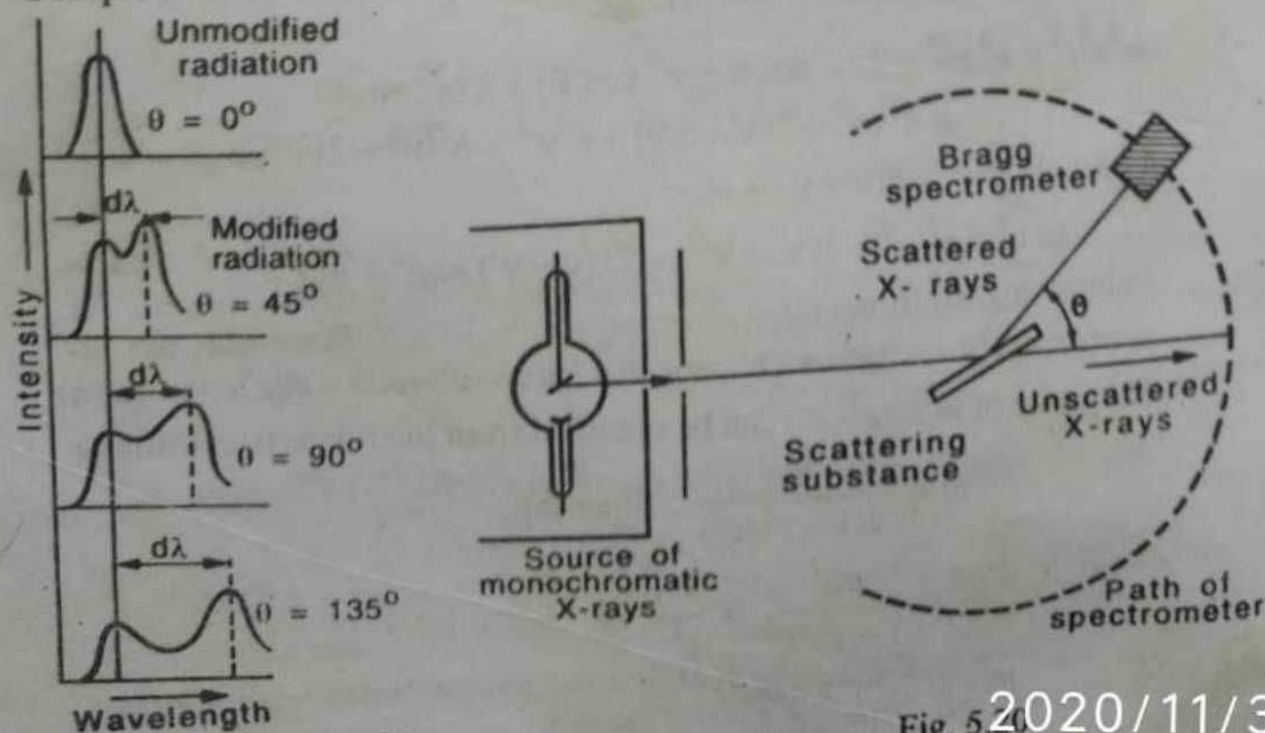

Fig. 5.21

Fig. 5.20

incident wavelength (λ), there exists a line of longer wavelength (λ'). The "Compton shift" $d\lambda$ is found to vary with the angle at which the scattered rays are observed.

Direction of Recoil electron : Dividing Eq. (5) by Eq. (4), we get

$$\tan \phi = \frac{h\nu' \sin \theta}{h(\nu - \nu' \cos \theta)} = \frac{\nu' \sin \theta}{(\nu - \nu' \cos \theta)} \qquad \ldots (12)$$

Using Eq. (10), we get

$$\frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1 - \cos \theta) = \frac{1}{\nu} + \frac{h}{m_0 c^2} \cdot 2 \sin^2 \frac{\theta}{2}$$

or $\quad \nu' = \dfrac{\nu}{1 + \left(\dfrac{h\nu}{m_0 c^2}\right) 2 \sin^2 \dfrac{\theta}{2}} = \dfrac{\nu}{1 + 2\beta \sin^2 \left(\dfrac{\theta}{2}\right)}$ where $\beta = \dfrac{h\nu}{m_0 c^2}$ $\ldots (13)$

Substituting this value of ν' in Eq. (12), we get

$$\tan \phi = \frac{\nu \sin \theta \Big/ \left[1 + 2\beta \sin^2 \left(\dfrac{\theta}{2}\right) \right]}{\left[\nu - \left\{ \nu \cos \theta \Big/ \left(1 + 2\beta \sin^2 \dfrac{\theta}{2} \right) \right\} \right]} = \frac{\cot \left(\dfrac{\theta}{2}\right)}{(1 + \beta)}$$

$\therefore \qquad \tan \phi = \dfrac{\cot \left(\dfrac{\theta}{2}\right)}{1 + \left(\dfrac{h\nu}{m_0 c^2}\right)}$ $\qquad \ldots (14)$

Kinetic Energy of Recoil electron : The K.E. of recoil electron is the difference between the energies of incident and scattered photons, i.e.,

$$\text{K.E.} = h\nu - h\nu'.$$

$$\text{K.E.} = h\nu - h\left[\frac{\nu}{1 + 2\beta \sin^2 (\theta/2)} \right] = h\nu \left[\frac{2\beta \sin^2 (\theta/2)}{1 + 2\beta \sin^2 (\theta/2)} \right] \qquad \ldots (15)$$

where $\beta = h\nu/m_0 c^2$

Example 1. X-rays of wavelength 0.7080 Å are scattered from a carbon block through an angle of 90° and are analysed with a calcite crystal, the interplanar distance of whose reflecting planes is 3.13 Å. Determine the angular separation, in the first order, between the modified and the unmodified rays.

Sol. Wavelength of the modified rays $= \lambda' = \lambda + \dfrac{h}{m_0 c}(1 - \cos \theta)$

$$= 0.7080 \times 10^{-10} \text{m} + \left(\frac{6.63 \times 10^{-34}}{9.11 \times 10^{-31} \times 3 \times 10^8} \right) \text{m} = 0.7323 \text{ Å}$$

Let θ and θ' be the angles of Bragg reflections corresponding to the wavelengths λ and λ'. Then, for $n = 1$ (first order),